

# ESTIMATING THE TARGET TRACKING USING A SET OF RANGE –PARAMETERS FOR GAIN MODIFICATION USING KALMAN FILTERS

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## **ABSTRACT**

*Bearings-only tracking using the modified gain extended Kalman filter (MGEKF) configured in Cartesian coordinate systems is reviewed. A new tracking approach is proposed which consists of a set of weighted MGEKFs each with a different initial range estimate and this is referred to as the range-parameterized (RP) tracker. This new approach overcomes the problems exhibited with existing MGEKF trackers.*

*Results are presented for a typical tracking scenario, involving a manoeuvring observer and a constant velocity target. The results show that the RP tracker gives stable, consistent and unbiased estimates in all the cases considered. Although only constant velocity target trajectories have been considered in this paper, the RP tracker provides a natural framework for consideration of manoeuvring targets.*

## **1. INTRODUCTION**

The objective of bearings-only tracking is to determine the trajectory of the target based on a time series of bearing measurements from a single observer. It is assumed that the motion of the target is constrained to be a straight line, constant-speed segments separated infrequently by manoeuvres in course and speed. The tested data in this paper are drawn from the sonar environment where we are only having the bearing information related to target which are used for finding the parameters like range, course and speed of the target. On the prior knowledge of the target motion, we are initializing the inputs of the target. However; the results are also applicable to the radar environment where a faster update rate compensates for the higher target speeds. The use of the extended Kalman filter for bearings- only tracking is a widely researched topic.

Here the paper is in passive sonar environment, with the bearing only information we are predicting the target position and direction. On the prior knowledge of the target motion, range parameterized tracker gives better solution, increases stability and accuracy. It is assumed that own ship manoeuvres in course and speed and target move in a straight line with a constant velocity. Initially error will be more while tracking the target, using MGEKF and RPEKF the error is reduced and tracks the target accurately, as we update the solution the error will be reduced gradually, slowly after some updates we get the desired solution and the solution converges. Kalman Filter is a minimum variance Estimator. It applies only to linear systems. (The measurements and state estimates must be linearly related).

The Kalman Filter is a computationally, highly efficient algorithm. Kalman Filter is defined as the minimum mean square estimator in recursive processing. The main aim is to reduce the variance that is diagonal elements of the covariance matrix. That's why kalman filter is the optimal estimator in the given situation. It can be applied to non-linear systems. The result is what is called "**Extended Kalman Filter**". However, most of the systems what we come across in our practical life are non-linear. The non-linearity can be associated either with the process model or with the observation model or with both. **Extended kalman filter** is similar to the kalman filter, it is applied to non linear systems, using EKF we approximate non linear systems into linear systems and the remaining process is almost similar to KF. Unlike its linear counterpart, the EKF is *not* an optimal estimator. In addition, if the initial estimate of the state is wrong, or if the process is modeled incorrectly, the filter may quickly diverge, owing to its linearisation. Another problem with the EKF is that the estimated covariance matrix tends to underestimate the true covariance matrix and therefore risks becoming inconsistent in the statistical sense without the addition of "stabilising noise".

Having stated this, the EKF can give reasonable performance, and is arguably the *de facto* standard in navigation systems and GPS. The fundamental flaw of EKF is that the distributions or densities of various random variables are no longer normal after undergoing their respective nonlinear transformations. The EKF is simply an adhoc state estimator that only approximates the optimality of Baye's rule by linearization. EKF Gain update equation is given by:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + V_k R_k V_k^T)^{-1} \dots(i)$$

An important feature of EKF is that the Jacobian  $H_K$  in equation for kalman gain  $K_K$  serves to correctly propagate or magnify only the relevant component of measurement information. If there is a one-to-one mapping between measurement and state via h Jacobian  $H_K$  affect Kalman gain. If in overall measurements there is no one-to-one mapping between measurement and state and the filter will quickly diverge. So the process is unobservable.

EKF exhibits unstable behaviour characteristics when applied to bearings-only target motion analysis. Anomalies such as premature covariance collapse and solution divergence have been observed even under favorable operating conditions. To circumvent these difficulties, **Modified Gain Extended Kalman Filter** (MGEKF) algorithm is used.

Since TMA (target motion analysis) estimation process is unobservable for constant own ship motion, it is not surprising that all components of solution vector typically contain bias errors prior to first own ship maneuver. However, once this maneuver requirement is satisfied, it is shown that only the estimated range vector will remain biased, the corresponding velocity estimates become asymptotically unbiased. It reveals that this residual range bias is highly dependent upon geometry and can be significantly altered by subsequent own ship maneuvers. Moreover, the optimum maneuver strategy for bias reduction is seen to simultaneously enhance overall tracking performance thus undesirable or inconsistent tactics are avoided. Therefore modified gain extended kalman filter (MGEKF) is used. EKF is difficult to tune and often gives unreliable estimates if system non-linearities are severe. This is because EKF depends on linearization to propagate the mean and covariance of state.

The problem with nonlinear system is that it is difficult to transform a probability density function through a general nonlinear function. We were able to obtain extract nonlinear transformations of mean and covariance, but only for a simple two dimensional transformation. The EKF works on the principle that a linearized transformation of means and covariances is

approximately equal to true nonlinear transformation but the approximation could be unsatisfactory.

## 2. MATHEMATICAL MODELLING:

### 2.1. Target Motion Parameters:

The derivation of the modified gain function of Song and Speyer's extended Kalman filter is slightly modified as follows. Let the target state vector be  $X_s(k)$

$$X_s(k) = [x(k) \quad y(k) \quad R_x(k) \quad R_y(k)] \text{---(ii)}$$

where  $x(k)$  and  $y(k)$  are target velocity components and,  $R_x(k)$  and  $R_y(k)$  are range components respectively.

The target state dynamic equation is given by eq

$$X_s(k+1) = \Phi X_s(k) + b(k+1) + \omega(k) \text{---(iii)}$$

where  $\Phi$  and  $b$  are transition matrix and deterministic vector respectively. The transition matrix is given by

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix}$$

Where  $t$  is sample time

$$b(k+1) = [0 \quad 0 \quad -\{x_o(k+1) - x_o(k)\} \quad -\{y_o(k+1) - y_o(k)\}]$$

$$\text{And } \Gamma = \begin{bmatrix} t & 0 \\ 0 & t \\ t^*t/2 & 0 \\ 0 & t^*t/2 \end{bmatrix}$$

Where  $x_o$  and  $y_o$  are observer position components. The plant noise,  $\omega(k)$  is assumed to be zero mean white Gaussian with  $E[\omega(k)\omega(j)] = Q \delta_{kj}$ . True North convention is followed for all angles to reduce mathematical complexity and for easy implementation. The bearing measurement,  $B_m$  is modeled as

$$B_m(k+1) = \tan^{-1}(R_x(k+1)/R_y(k+1)) + \zeta(k) \text{---(iv)}$$

Where  $\zeta(k)$  is error in the measurement and this error is assumed to be zero mean Gaussian with variance. The measurement and plant noises are assumed to be uncorrelated to each other. eq. below is a non-linear equation and is linearized by using the first term of the Taylor series for  $R_x$  and  $R_y$ . The measurement matrix is obtained as

$$H(k+1) = \begin{bmatrix} 0 & 0 & \frac{\Lambda}{R_y(k+1/k)} \frac{\Lambda^2}{(k+1/k)} & -\frac{\Lambda}{R_x(k+1/k)} \frac{\Lambda^2}{(k+1/k)} \end{bmatrix} \text{---(v)}$$

Since the true values are unknown, the estimated values of  $R_x$  and  $R_y$  are used in (presented in above equation). The covariance prediction is

$$P(k+1/k) = \Phi(k+1/k)P(k/k)\Phi^T(k+1/k) + \Gamma Q(k+1)\Gamma^T \text{---(vi)}$$

The Kalman gain is

$$G(k+1) = P(k+1/k)H^T(k+1)[\sigma^2 + H(k+1)P(k+1/k)H^T(k+1)]^{-1} \text{ ---(vii)}$$

The state and its covariance corrections are given by

$$X(k+1/k+1) = X(k+1/k) + G(k+1)[B_m(k+1) - h(k+1, X(k+1/k))] \text{ ----(viii)}$$

Where  $h(k+1, X(k+1/k))$  is the bearing using predicted estimate at time index  $k+1$ ,

$$P(k+1/k+1) = [1 - G(k+1)g(B_m(k+1), X(k+1/k))]P(k+1/k) \text{ --(ix)}$$

$$[1 - G(k+1)g(B_m(k+1), X(k+1/k))]^T + \sigma^2 G(k+1)G^T(k+1)$$

Where  $g(\cdot)$  is modified gain function. The value of  $g$  is

$$g = \begin{bmatrix} 0 & 0 & \cos B_m \wedge / (R_x \sin B_m + R_y \cos B_m) & -\sin B_m \wedge / (R_x \sin B_m + R_y \cos B_m) \end{bmatrix} \text{ ---(X)}$$

Since the true bearing is unavailable in practice, it is replaced by the measured bearing to compute the function-  $g(\cdot)$ .

## 2.2. RANGE PARAMETERIZED (RP) tracker

The new tracking approach proposed in this project is to commence tracking with a number of independent EKFs, each with a different initial range estimate. At each update the filters are weighted for their consistency with the measured bearing. After a number of updates the likelihood of some of the filters will fall below a threshold and will no longer be processed. How quickly this happens depends on the scenario geometry, the observer and target trajectories and the number and type of observer manoeuvres. In good tracking conditions, the correct filter will dominate very quickly and within a short time it will be the only filter being processed.

Here we are using set of independent filters, with initializing different scenarios; it is observed that the tracking accuracy depends on the initializing scenario. RP tracker is used to give stable tracking performance in the majority of conditions. In principle the methodology of the **RP** tracker could be applied to any coordinate system (Cartesian, polar and variants of modified polar). However, the number of cells into which the range has to be parameterized in order to give acceptable tracking performance varies according to the coordinate system in which it is implemented.

### Updating the weights

The state and covariance estimates of each independent EKF tracker are updated at each bearing measurement. In addition, the cell weightings are updated using Bayer's theorem, based on the assumption that the forecast and measured bearings are Gaussian with zero mean error. The basic idea is to use a number of independent EKF trackers in parallel, each with a different initial range estimate. To do so, the range interval of interest is divided into a number of subintervals, and each subinterval is dealt with an independent EKF. To determine how the state estimates of parallel filters are combined, we need to compute the weights associated with each EKF.

The weight of filter  $i$  at time  $k$  is given by: 
$$w_k^i = \frac{p(z_k / i)w_{k-1}^i}{\sum_{j=1}^N p(z_k / j)w_{k-1}^j} \text{ ---- (xi)}$$

Where  $p(z_k / i)$  is the likelihood of measurement  $z_k$ , given that the target originated in subinterval  $i$ . If one uses  $N$  parallel filters, in the absence of prior information about the true

target range, all initial weights  $w$  are set to  $1/N$ . Typically; in the RP-EKF one uses EKFs in the Cartesian coordinates. Then, assuming Gaussian statistics, the likelihood can be computed as

$$p(z_k / i) = \frac{1}{\sqrt{2\pi\sigma_i^2} \{\exp[-0.5(z_k - z_{k-1}^i / \sigma_i)^2]\}} \text{--- (xii)}$$

The exponential term ensures that the cell receives low weighting because of large variation between the forecast and measured bearing compared with the combined standard deviation. Conversely, a small difference implies much better tracking and the cell receives a higher weighting. The speed with which the cell containing the true target position approaches a weighting of unity depends on the degree of range observability in the tracking scenario being considered. In a good tracking scenario the correct cell will dominate very quickly.

### 3. Results

ERROR THAT IS TOLERATED:

- ±10%, in range estimate.
- ± 0.5 DEG, in bearing estimate.
- ± 5 deg, in course estimate.
- ± 2 knots, in velocity.

Table 5.1: True and Predicted Values of Target

Converge time (seconds)	True range (meters)	Predicted Range (meters)	Target bearing (deg)	Predict bearing (deg)
150	3705.24	3753.30	66.49	66.76

Table 5.2: Minimum Errors of The Scenario

Scenario	Min range error (meters)	Min bearing error (deg)	Min course error (deg)	Min speed error (knots)
1	0.0020	0.0000	0.0001	0.0000

Target course (deg)	predict course (deg)	target velocity (knots)	Predict Velocity (knots)
120.00	123.91	15.4	15.35

DURATION OF RUN IS 800 SECONDS

SCENARIO	
RANGE	4000.00(MTS)
BEARING	45.00(DEG)
TCR	120.00(DEG)
SIGMAB	0.33
VT	15.45(KNOTS)
VO	10.30(KNOTS)

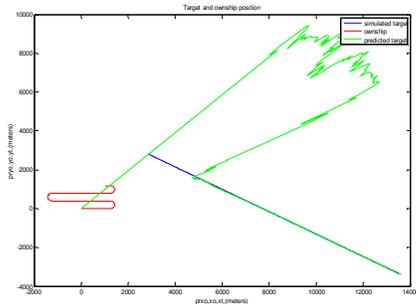


Fig.1. Tracking Using RPEKF

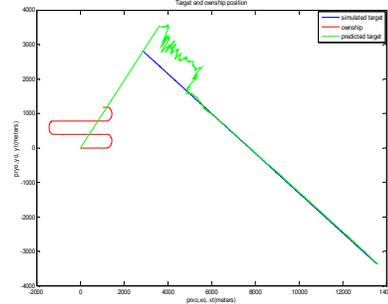


Fig. 2. Tracking Using MGEKF

For the scenario: RPEKF converges faster than MGEKF. So, RPEKF gives better solution than MGEKF

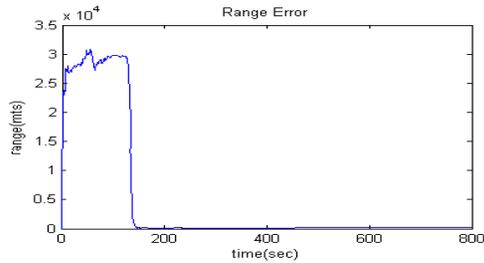


Fig. 3. Range Error

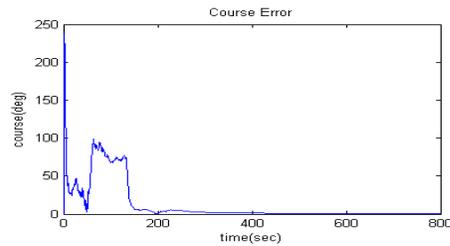
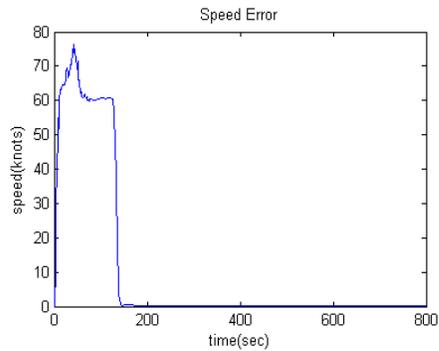
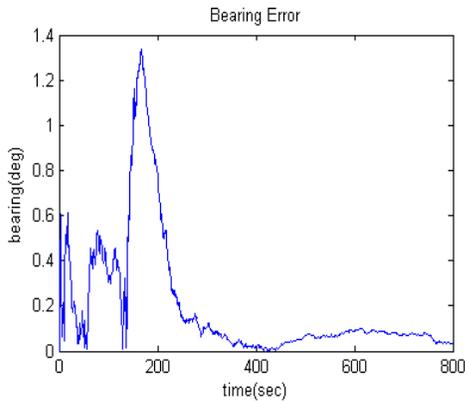


Fig. 4. Bearing Error

Fig. 5. Course Error

Fig. 6. Bearing Error



## 4. Conclusion

The new tracking approach, referred in this paper as the range-parameterized (**RP**) tracker which gives considerably better tracking performance than the MGEKF. In particular, the **RP** tracker gives significantly lower range errors than the other trackers. The major advantage of the **RP** tracker over the **MGEKF** trackers is that, it divides the large prior range uncertainty region into a number of smaller cells. This provides the prior knowledge of the target velocity, this prevents tracker instability and can allow the range to be inferred even before the first observer manoeuvres. In this work, a passive sonar environment with the bearing only information is considered by predicting the target position and direction. On the prior knowledge of the target motion, range parameterized tracker gives better solution, increases stability and accuracy.

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