THE EFFECT OF MASS TRANSFER WITH MIXED CONVECTION BOUNDARY LAYER FLOW ALONG VERTICAL MOVING THIN NEEDLES WITH VARIABLE HEAT FLUX

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ABSTRACT

The study of the steady laminar mixed convection boundary layer flow of an incompressible viscous fluid, with heat and mass transfer along vertical thin needles with variable heat flux has been considered. The governing boundary layer equations are first transformed into non dimensional form and then after using similarity transformations converted into set of ordinary differential equations. The set of ordinary differential equations are solved with the help Runge-Kutta method with shooting technique. The value of skin friction coefficient and the surface temperature, wall concentration, velocity profile as well as temperature profile are obtained for different values of dimensionless parameters with m=0, which is case for a blunt nosed needle. The magnitude of velocity decreases with increasing needle size and increases with increasing local buoyancy parameter. The graphical and tabulated results are presented.

KEY-WORDS

Mixed Convection, Variable Heat Flux, Similarity Transformation, Runge-Kutta Method, Shooting Technique.

1. INTRODUCTION

Boundary layer flow on a moving surface is important in number of engineering processes. The flow over thin needles has considerable importance because the measuring devices, such as a hot wire anemometer or a shielded thermocouple, are often a very thin wire or needle. Therefore, the detailed analysis of the flow over such slender needle shaped body is of considerable practical interest.

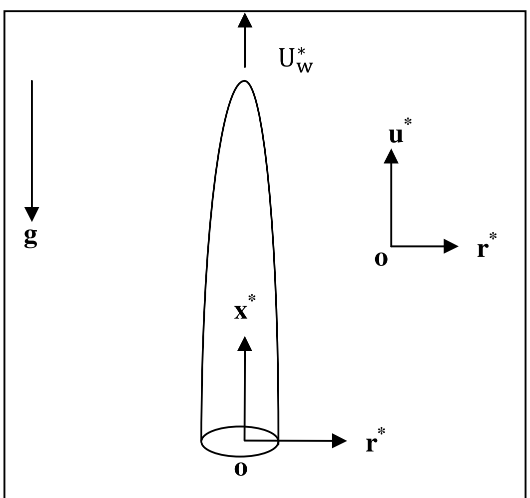
The mixed convection boundary layer flow over a vertical needle has been studied by many researchers. Chen (1983) studied the natural convection from needles with variable wall heat flux. In 1987 Chen studied the mixed convection flow about slender bodies of revolution. Wang (1989) studied the problem of free and mixed convection boundary layer flows on a vertical adiabatic thin needle with a concentrated heat source at the tip of the needle. It was found from the studies mentioned above that there is a significant influence of the shape, size and wall heat flux variations of the thin needles upon the flow and heat transfer characteristics. Rosenhead presented an analysis for the momentum boundary layer flow of a Newtonian fluid over a static thin cylinder. Agarwal have investigated numerically the momentum and thermal boundary layers for power law fluids over a moving thin needle. After that the problem of steady free and mixed convection flow from a static thin vertical cylinder has been studied by Gorla. Ahmad (2008) studied the steady laminar boundary layer flow over moving thin needles with variable surface heat flux.

One of the limitations of the above problems is neglection of the diffusion equation. However, in many real boundary layer flows, the flow and the heat transfer and mass diffusion are coupled. Many researchers also studied the heat and mass transfer problem along the moving cylinder due to its wide practical application. Chamkha (2011) studied the problem of heat and mass transfer along a permeable linearly moving cylinder with heat generation/absorption, chemical reaction and suction/injection effects in the presence of uniform transverse magnetic field. Gnaneswara Reddy Machireddy (2013) presented the study of chemically reactive species and radiation effects on MHD convective flow past a moving vertical cylinder. Sharma and Konwar (2015) presented the study of heat and mass transfer due to axially moving cylinder in presence of thermal diffusion, radiation and chemical reactions in a binary fluid mixture.

No analysis seems to have been presented the study of effect of mass transfer with mixed convection boundary layer flow along vertical moving thin needle with variable heat flux. Due to the importance of this study in many applications, the present study is attempted. The present study deals with the study of the effect of mass transfer with mixed convection boundary layer flow along vertical moving thin needle with variable heat flux. The governing fluid flow equations are transformed into transformed into non dimensional form and then after using similarity transformations converted into set of ordinary differential equations. The set of ordinary differential equations are solved with the help Runge-Kutta method with shooting technique. The value of skin friction coefficient and the surface temperature, wall concentration, velocity profile as well as temperature profile are obtained for different values of dimensionless parameters with m=0, which is case for a blunt nosed needle.

2. FORMULATION OF PROBLEM

The steady laminar boundary layer flow of an incompressible viscous fluid over a moving vertical thin needle with variable heat flux and with mass transfer has considered. In figure, x* and r* are the axial and radial co-ordinates, u* and v* are the velocity components along the x* and r* axes, $U_w^*(x^*)$ is the velocity of moving needle and g is acceleration due to gravity. Under the Boussinesq approximations the governing fluid flow equations are



Figure

$$\frac{\partial(r^* u^*)}{\partial x^*}+\frac{\partial(r^* v^*)}{\partial r^*}=0 \qquad \ldots\ldots(1)$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial r^*}=\frac{\nu}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial u^*}{\partial r^*}\right)+g\beta_t(T^*-T_\infty)+g\beta_c(C^*-C_\infty) \qquad \ldots\ldots(2)$$

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial r^*}=\frac{\alpha}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial T^*}{\partial r^*}\right) \qquad \ldots\ldots(3)$$

$$u^*\frac{\partial C^*}{\partial x^*}+v^*\frac{\partial C^*}{\partial r^*}=\frac{D}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial C^*}{\partial r^*}\right) \qquad \ldots\ldots(4)$$

The boundary conditions are

$$v^*=0, \qquad u^*=U_w{}^*(x^*), \qquad k\frac{\partial T^*}{\partial r^*}=-q_w{}^*(x^*), \qquad d\frac{\partial C^*}{\partial r^*}=-d_w{}^*(x^*), \qquad \text{at } r^*=R^*(x^*)$$

$$u^*=0, \qquad T^*=T_\infty, \qquad C^*=C_\infty \qquad \text{as } r^*\to\infty \qquad \ldots\ldots(5)$$

where T* and C* are the local fluid temperature and local fluid concentration; $T\infty$ is ambient fluid temperature; βt and βc are coefficient of thermal expansion and coefficient of mass transfer expansion respectively; α is thermal diffusivity; D is binary diffusion coefficient; ν is kinematic viscosity; d is diffusivity of fluid. The radius of vertical slender paraboloid needle is described as $r^*=R^*(x^*)$. $q_w{}^*(x^*)$ is variable surface heat flux and $d_w{}^*(x^*)$ is variable surface mass flux. The following non dimensional transformations have been used to convert the equations (1) - (4) with boundary conditions (5) into non dimensional form,

$$x=\frac{x^*}{L}, r=Re^{1/2}\left(\frac{r^*}{L}\right), \qquad R(x)=Re^{\frac{1}{2}}\left(\frac{R^*(x^*)}{L}\right), \qquad u=\frac{u^*}{U_0}, \qquad v=Re^{1/2}\left(\frac{v^*}{U_0}\right),$$

$$U_w(x)=\frac{U_w{}^*(x^*)}{U_0}, \quad T=\frac{kRe^{\frac{1}{2}}(T^*-T_\infty)}{q_0 L}, \qquad C=\frac{dRe^{\frac{1}{2}}(C^*-C_\infty)}{d_0 L}, \quad v_0=\frac{v_0^*}{v_0}Re^{\frac{1}{2}}$$

$$q_w(x)=\frac{q_w^*(x^*)}{q_0}, \qquad d_w(x)=\frac{d_w^*(x^*)}{d_0} \qquad \ldots\ldots(6)$$

Thus obtained transformed non dimensional equations are:

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0 \qquad \ldots\ldots(7)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+T\left(\frac{Gr}{Re^{\frac{5}{2}}}\right)+C\left(\frac{Gm}{Re^{\frac{5}{2}}}\right) \qquad \ldots\ldots(8)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{1}{r\,Pr}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \qquad \ldots\ldots(9)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial r}=\frac{1}{r\,Sc}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right), \qquad \ldots\ldots(10)$$

where Pr is Prandtl number, Sc is Schmidt number, Gr is Grashof number, Gm is Grashof number for mass diffusion and Re is Reynolds number.

The transformed boundary conditions are:

$$u = U_w(x), \qquad v = 0, \qquad \frac{\partial T}{\partial r} = -q_w(x), \qquad \frac{\partial C}{\partial r} = -d_w(x), \quad \text{at } r = R(x)$$

and

$$u = 0, \qquad T = 0, \qquad C = 0 \qquad \text{as } r \to \infty \qquad \ldots\ldots(11)$$

Now the used similarity transformations are:

$$\eta = x^{m-1}r^2, \quad \psi = xf(\eta), \quad U_w(x) = x^m, \quad q_w(x) = x^{(5m-3)/2}, \quad d_w(x) = x^{(5m-3)/2}$$

$$T = x^{m-1}\theta(\eta), \quad C = x^{m-1}\emptyset(\eta), \qquad \lambda = \frac{g\beta\left(\frac{q_0L}{k}\right)L}{U_0^2 Re^{\frac{1}{2}}} = \frac{Gr}{Re^{\frac{5}{2}}},$$

$$\lambda_c = \frac{g\beta_t\left(\frac{d_0L}{d}\right)L}{U_0^2 Re^{\frac{1}{2}}} = \frac{Gm}{Re^{\frac{5}{2}}} \qquad \ldots\ldots(12)$$

Here λ is mixed convection parameter and $\frac{\lambda_c}{\lambda} = N$ is known as local buoyancy ratio parameter, η is similarity variable, ψ is stream function, $\emptyset$ is dimensionless concentration and θ is dimensionless temperature. Also d_0 and q_0 are characteristic mass flux and characteristic heat flux respectively.

Expression for η gives the shape and size of the body, when we put η=a in the expression we get dimensionless needle radius R(x).

$$R(x) = a^{\frac{1}{2}}x^{\frac{1-m}{2}} \qquad \ldots..(13)$$

Here 'a' is dimensionless needle size.

$$u = \frac{1}{r}\left(\frac{\partial\psi}{\partial r}\right), \qquad v = -\frac{1}{r}\left(\frac{\partial\psi}{\partial x}\right)$$

After introducing these transformations in equations (8)-(10) and boundary conditions (11), we get following ordinary differential equations:

$$8\left(\eta f''\right)' + 4ff'' - 4mf'^2 + \lambda(\theta + N\emptyset) = 0 \qquad \ldots\ldots(14)$$

$$\frac{2}{Pr}\left(\eta\theta'\right)' + f\theta' - (2m-1)f'\theta = 0 \qquad \ldots\ldots(15)$$

$$\frac{2}{Sc}\left(\eta\emptyset'\right)' + f\emptyset' - (2m-1)f'\emptyset = 0 \qquad \ldots\ldots(16)$$

and transformed boundary conditions,

$$f'(a) = \frac{1}{2}, \quad f(a) = \frac{(1-m)a}{2}, \quad \theta'(a) = -\frac{1}{2}a^{-\frac{1}{2}}, \quad \emptyset'(a) = -\frac{1}{2}a^{-\frac{1}{2}} \quad \text{at } \eta = a$$

$$f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \emptyset(\infty) = 0 \qquad \text{as } \eta \to \infty \qquad \ldots\ldots(17)$$

The surface skin friction coefficient Cf, the surface temperature Tw and surface concentration Cw, are defined as,

$$C_f = \frac{\tau_w}{\rho[U_w{}^*(x^*)]^{\frac{1}{2}}}, \qquad T_w = \frac{kRe^{\frac{1}{2}}(T_w^* - T_\infty)}{q_0L}, \qquad C_w = \frac{dRe^{\frac{1}{2}}(C_w^* - C_\infty)}{d_0L}$$

Where τ_w is the skin - friction given by,

$$\tau_w = \mu\left(\frac{\partial u^*}{\partial r^*}\right)_{r^*=R^*(x^*)}$$

Using non-dimensional transformations and similarity transformation finally we get,

$$C_fRe_x^{1/2} = 4a^{\frac{1}{2}}f''(a), \quad T_w = x^{2m-1}\theta(a), \quad C_w = x^{2m-1}\emptyset(a)$$

Here $Re_x = \frac{U_w{}^*(x^*)x^*}{\nu}$ is the local Reynolds number.

3. RESULTS AND DISCUSSION

The system of coupled ordinary differential equations (14)-(16) with boundary conditions (17) are solved with the help of Runge-Kutta method with shooting technique. Results have been obtained for different values of parameters. The values of Pr are taken from 0.7 to 7 and values of Schmidt number are taken from 0.5 to 1.3. The dimensionless needle size is varying from 0.02 to 0.1. The values of local buoyancy parameter are considered from 0.5 to 2.

Figures (1), (2) and (3) show velocity profile, temperature profile and concentration profile respectively, for varying values of Schmidt number, keeping other parameters constant (Pr=0.7, N=1, a=0.1, λ=5). It have been seen that magnitude of velocity (f') and momentum boundary layer thickness both decreases with increasing values of Sc (0.5, 0.9, 1.0, 1.3). In figure (2) magnitude of temperature (θ) decreases, but thermal boundary layer thickness increases with increasing values of Sc. Figure (3) depicts that magnitude of concentration (Φ) and concentration boundary layer thickness both decrease with increasing values of Sc. Figures (4), (5) and (6) show the effect of mixed convection parameter (λ) on velocity, temperature and concentration profiles respectively. Figures (4) and (5) represents that magnitude of velocity and temperature both increases, but momentum boundary layer thickness and thermal boundary layer thickness both decreases with increasing values of λ (λ=2,4,5). Figure (6) shows that concentration boundary layer thickness decreases with increasing values of λ. Figures (7)-(9) show the effects of variation of Prandtl number on velocity, temperature and concentration profiles respectively. These figures show that magnitude of velocity and temperature both decreases while magnitude of concentration increases with increasing values of Prandtl number (Pr). Figures (10) and (11) show the effects of dimensionless needle size and local buoyancy parameter on velocity profile respectively. There is no significant change have been observed in temperature and concentration profiles with varying dimensionless needle size (a) and local buoyancy parameter (N). Figure (10) shows that magnitude of velocity decreases with increasing value of a, and figure (11) gives that magnitude of velocity increases with increasing values of N.

Table 1 gives the values of skin friction coefficient, temperature at wall of the needle and concentration at the wall of needle surface with varying dimensionless needle size and with local buoyancy parameter. It has been seen that $f''(a)$, $\theta(a)$ and $\Phi(a)$ increases with increasing values of needle size. But magnitude of $f''(a)$ decreases with increasing values of N and there is no significant change in $\theta(a)$ and $\Phi(a)$ due to change in N. It has been noticed from Table 2 that with increasing values of Pr and Sc, magnitude of $f''(a)$, $\theta(a)$ and $\Phi(a)$ decreases. Table 3 shows the effect of mixed convection parameter (λ) on $f''(a)$, $\theta(a)$ and $\Phi(a)$, which shows that values of $f''(a)$, $\theta(a)$ and $\Phi(a)$ decrease with increasing values of λ.

Table 1: Values of Skin friction coefficient, temperature and concentration at the wall with varying dimensionless needle size (a) and local buoyancy parameter (N)

Pr=0.7, λ=5, N=1, m=0, Sc=1				Pr=0.7, λ=5, a=0.1, m=0, Sc=1			
a	$f''(a)$	$\theta(a)$	$\Phi(a)$	N	$f''(a)$	$\theta(a)$	$\Phi(a)$
0.02	173.0678	17.8781	13.9554	0.5	28.6768	8.3021	6.3098
0.04	85.4902	12.9901	9.8959	1.1	34.9441	8.4398	6.3098
0.06	57.0845	10.8897	8.0991	1.2	35.9019	8.4398	6.3098
0.09	37.6718	8.8439	6.6239	1.5	38.8041	8.4909	6.3098
0.10	33.9758	8.4398	6.3098	2.0	43.4261	8.5599	6.3098

Table 2 : Values of Skin friction coefficient, temperature and concentration at the wall with varying Prandtl number (Pr) and Schmidt number (N)

a=0.1, λ=5, N=1, m=0, Sc=1				a=0.1, λ=5, N=1, m=0, Pr=0.7			
Pr	$f''(a)$	$\theta(a)$	$\Phi(a)$	Sc	$f''(a)$	$\theta(a)$	$\Phi(a)$
0.7	33.9758	8.4398	6.3098	0.5	45.2196	8.9809	10.5721
2.0	21.7032	3.1888	6.2004	0.9	35.4191	8.4609	6.9678
4.0	18.6701	1.5897	6.1451	1.0	33.9758	8.4398	6.3098
7.0	17.6612	0.93003	6.1398	1.3	30.8728	8.3509	4.8698

Table: 3 Values of Skin friction coefficient, temperature and concentration at the wall with varying mixed convection parameter (λ)

Pr=0.7, a=0.1, N=1, m=0, Sc=1			
λ	$f''(a)$	$\theta(a)$	$\Phi(a)$
2	15.8008	6.9878	6.1788
4	28.5408	8.2348	6.2788
5	33.9758	8.4398	6.3098

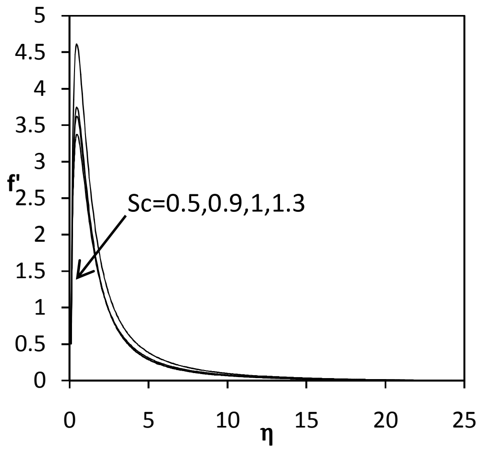


FIGURE 1
Velocity Profile for varying Sc
(Pr=0.7, N=1, λ=5, a=0.1)

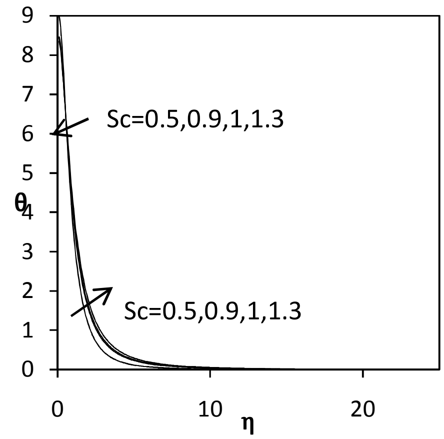


FIGURE 2
Temperature Profile for varying Sc
(Pr=0.7, N=1, λ=5, a=0.1)

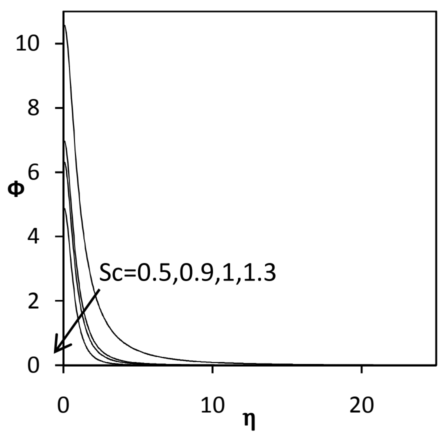


FIGURE 3
Concentration Profile for varying Sc
(Pr=0.7, N=1, λ=5, a=0.1)

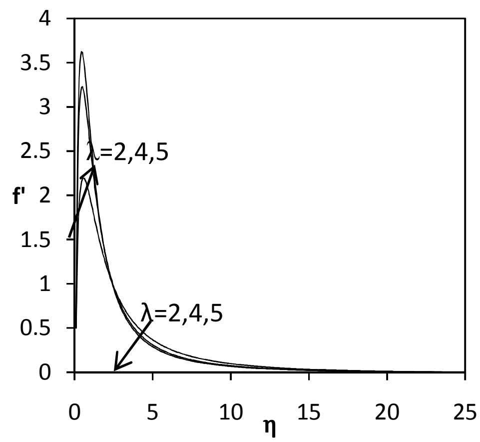


FIGURE 4
Velocity profile for varying λ
(Pr=0.7, N=1, a=0.1, Sc=1)

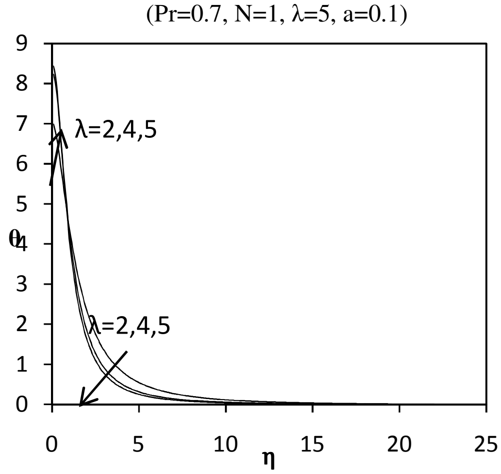


FIGURE 5
Temperature profile for varying λ
(Pr=0.7, N=1, Sc=1, a=0.1)

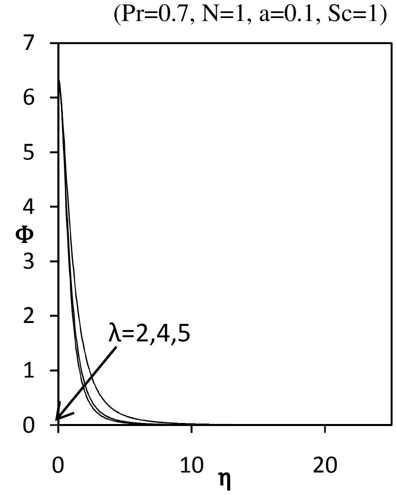


FIGURE 6
Concentration profile for λ
(Pr=0.7, N=1, Sc=1, a=0.1)

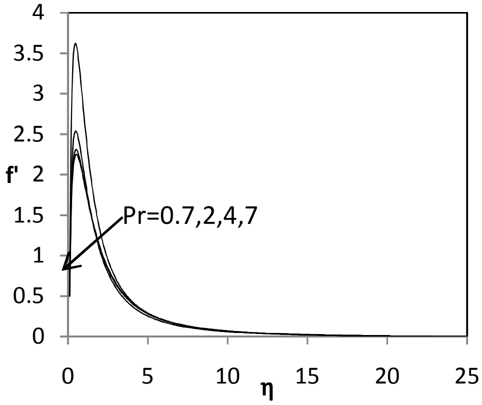


FIGURE 7
Velocity profile for varying Pr
(N=1, Sc=1, a=0.1, λ=5)

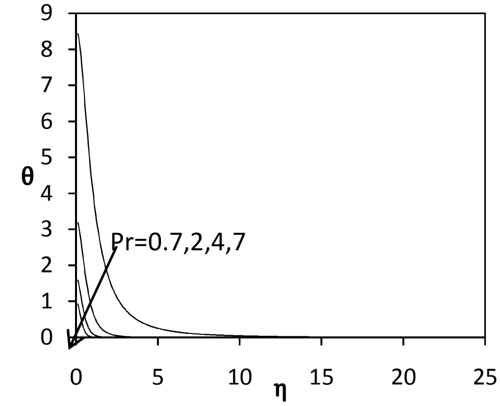


FIGURE 8
Temperature profile for Pr
(N=1, Sc=1, a=0.1, λ=5)

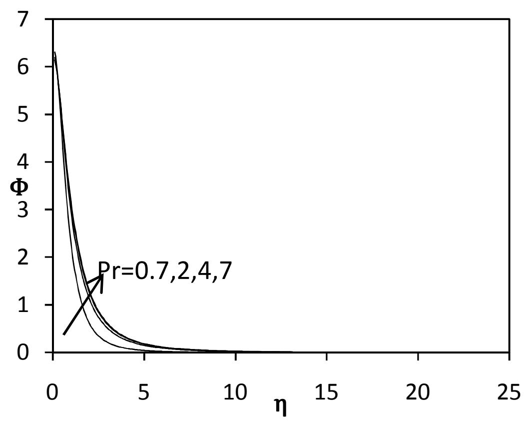


FIGURE 9
Concentration profile for varying Pr
(Sc=1, N=1, a=0.1, λ=5)

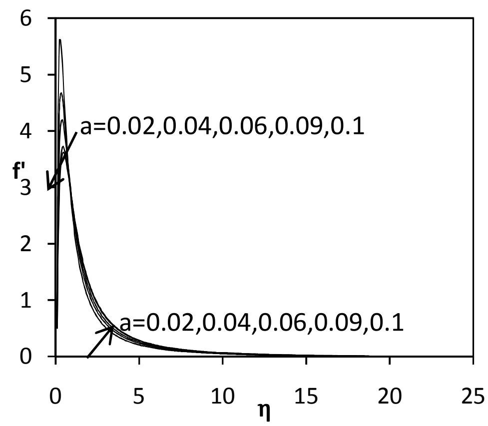


FIGURE 10
Velocity profile for varying 'a'
(Pr=0.7, N=1, Sc=1, λ=5)

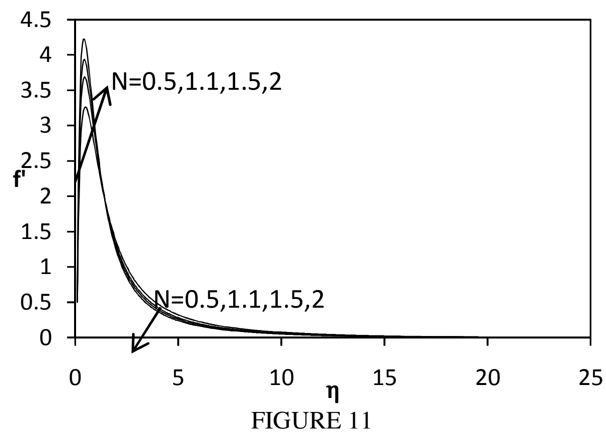


FIGURE 11
Velocity profile with varying N
(Pr=0.7, λ=5, Sc=1, a=0.1)

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