PEST CONTROL IN JATROPHA CURCAS

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ABSTRACT

This paper presents a mathematical framework involving bifurcation analysis and multiobjective nonlinear model predictive control for two models of pest control in Jatropha curcas plantations. Jatropha curcas is a biofuel crop that has the potential to replace fossil fuels in the near future. Jatropha plantations are expanding rapidly. Bifurcation analysis is a powerful mathematical tool used to address the nonlinear dynamics of any process. Several factors must be taken into account, and multiple objectives must be achieved simultaneously. The MATLAB program MATCONT was utilized to conduct the bifurcation analysis. The MNLMPC calculations were performed using the optimization language PYOMO in conjunction with the advanced global optimization solvers IPOPT and BARON. The bifurcation analysis revealed the presence of branch points in both models. These branch points are advantageous as they allow the multiobjective nonlinear model predictive control calculations to converge to the Utopia point, which represents the most beneficial solution. The combination of bifurcation analysis and multiobjective nonlinear model predictive control for pest control models in Jatropha curcas plantations is the primary contribution of this paper.

KEYWORDS

information, dissemination, bifurcation, optimization, control

1. BACKGROUND

Bhattacharyya et al (2006)[1] investigated pest control through viral disease using mathematical modeling and analysis. Manoharan et al (2006)[2] , discussed the emerging pest status of jatropha cucas l Shankar et al (2006)[3] , C., investigated the potential for the management of the insect pest of Jatropha curcas. Van den Bosch et al (2007)[4] researched theoretically disease control and its selection for damaging plant virus strains in vegetatively propagated staple food crops. Demirbas(2008)[5] studied various biofuel sources, biofuel policy, biofuel economy and global biofuel projections. Devi et al (2008)[6] , conducted studies on insect pest of Jatropha. Ndong et al (2009)[7] studied the life cycle assessment of biofuels from jatropha curcas in west Africa. Ghosh and Bhattacharya(2010)[8] conducted optimization studies in microbial pest control. Tang et al (2010)[9] performed a dynamical analysis of plant disease models with cultural control strategies and economic thresholds. Pandey et al (2012)[10] looked at Jatropha curcas as a potential biofuel plant for sustainable environmental development. Terren et al (2012)[11] studied principal disease and insect pests of *Jatropha curcas* in the lower valley of the Senegal River. Roy et al (2014)[12] studied the effect of mass transfer kinetics for maximum production of biodiesel from Jatropha Curcas Oil. Federico et al (2014)[13] performed conceptual and mathematical modeling of insect-borne plant diseases. Roy et al (2015)[14] mathematically studied the effect of insecticide spraying on Jatropha curcas plants to control mosaic virus. Roy(2015)[15] , looked at renewable energy biodiesel from ecology to production using a mathematical approach. Venturino et al (2016)[16] developed a model for Jatropha curcas plantations affected by the mosaic virus. Chowdhury et al (2016)[17] used a mathematical approach to study pest control for jatropha curcas plant through viral disease. Basir et al

(2016)[18] studied the effect of an awareness program for controlling mosaic disease in Jatropha curcas plantations. Basir and co-workers(2017,2018,2019)[20,2] performed theoretical studies regarding the control of pests that affect Jatropha curcas plantations. Chowdhury et al (2019)[22] developed a mathematical model for pest management in Jatropha curcas with integrated pesticides.

This paper aims to perform bifurcation analysis in conjunction with multiobjective nonlinear model predictive control (MNLMPC) for two pest management models in *Jatropha curcas* with pesticides. The two models that will be used are described in Chowdhury et al (2019)[22] and Roy et al (2015)[14]. This paper is organized as follows. First, the model equations for both models are presented. The numerical procedures (bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC) are then described. This is followed by the results and discussion, and conclusions.

2. MODEL EQUATIONS

The two models that will be used are those described in Chowdhury et al (2019)[22](Model 1) and Roy et al (2015)[14](Model 2).
In the first model (Model 1), J(t) represents the biomass of the Jatropha Curcas plant, Ps(t) is the susceptible pest, Pi(t) is the infected pest, and V(t) represents the bio-pesticides. The model equations are

$$\begin{aligned}
\frac{dJ}{dt} &= rJ\left(1-\left(\frac{J}{kc}\right)\right)-\lambda J(Ps)\\
\frac{dPs}{dt} &= mpar(\lambda JPs)\left(1-\left(\frac{(Ps+Pi)}{Ck}\right)\right)-\left(\alpha(PsV)\right)-\left(d1(u1Ps)\right)\\
\frac{dPi}{dt} &= \left(\alpha PsV\right)-\left(\xi Pi\right)-\left(d2(u1Pi)\right)\\
\frac{dV}{dt} &= \left(1-u2\right)\pi_v+ks(\xi Pi)-\left(\mu_v V\right)-\left(\alpha_1(PsV)\right)
\end{aligned} \qquad (1)$$

The model base parameter values are

$$\begin{aligned}
&\lambda = 0.0012; r = 0.05; \xi = 0.3; \mu_v = 0.1; d_1 = 0.6; d_2 = 0.8;\\
&kc = 50; \pi_v = 5; \alpha = 0.0025; alfa_1 = 0.003; mpar = 8; ks = 5; c = 0.003;\\
&ck = c*kc = 0.15; u1 = 0.5; u2 = 0.5;
\end{aligned}$$

u1 and u2 are the control variables.

In the second model (Model 2), the variable representing the healthy Jatropha plant is x(t), the latent plant is l(t), y(t) represents the infected plant, the healthy vector is u(t), and the infected vector is v(t). The model equations are

$$\begin{aligned}
\frac{dx}{dt} &= rx\left(1-\left(\frac{(x+l+y)}{k}\right)\right)-(k1(xv))+(\delta l) \\
\frac{dl}{dt} &= (k1(xv))-(al)-(\delta l) \\
\frac{dy}{dt} &= (al)-(gy)-(\beta y) \\
\frac{du}{dt} &= b(u+v)\left(1-\left(\frac{u+v}{mpar(x+l+y)}\right)\right)-(k2yu)-(cu)-(\gamma u) \\
\frac{dv}{dt} &= (k2yu)-(cv)-(\gamma v);
\end{aligned} \quad (2)$$

The model base parameter values are

$$r = 0.05; k = 0.5; k1 = 0.001; \delta = 8.541e-04; mpar = 300;$$
$$a = 0.5; g = 0.03; \beta = 0.003; b = 0.8; k2 = 0.008; c = 0.12; \gamma = 0.5$$

γ is the control variable.

3. BIFURCATION ANALYSIS

Bifurcation analysis is performed using the MATLAB software MATCONT Bifurcation analysis deals with multiple steady-states and limit cycles. Branch and limit points cause multiple steady states and Hopf bifurcation points cause limit cycles . MATCONT(Dhooge Govearts, and Kuznetsov, 2003[23]; Dhooge Govearts, Kuznetsov, Mestrom and Riet, 2004[24]) locates branch(BP) limit (LP) and Hopf(H) bifurcation points.

For an ODE system

$$\frac{dx}{dt} = f(x,\alpha) \quad (3)$$

$x \in R^n$ and α the bifurcation parameter, the tangent plane at any point $w = [w_1, w_2, w_3, w_4, ...w_{n+1}]$ must satisfy

$$Aw = 0 \qquad (4)$$

Where A is

$$A = [\partial f / \partial x \quad | \partial f / \partial \alpha] \qquad (5)$$

where $\partial f / \partial x$ is the Jacobian matrix. For both limit and branch points, the matrix $[\partial f / \partial x]$ must be singular. The n+1 th component of the tangent vector w_{n+1} = 0 for a limit point (LP)and for a branch point (BP) the matrix $\begin{bmatrix} A \\ w^T \end{bmatrix}$ must be singular. At a Hopf bifurcation point,

$$\det(2 f_x(x,\alpha) @ I_n) = 0 \qquad (6)$$

@ indicates the bialternate product while I_n is the n-square identity matrix. Hopf bifurcations cause limit cycles and should be eliminated because limit cycles make optimization and control tasks very difficult. Further details can be found in Kuznetsov (1998[37]; 2009[25,26]) and Govaerts [2000] [27]

4. MULTI OBJECTIVE NONLINEAR MODEL PREDICTIVE CONTROL (MNLMPC)

Flores Tlacuahuaz et al (2012)[28] developed a multiobjective nonlinear model predictive control (MNLMPC) method. This procedure is used for performing the MNLMPC calculations. Here $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ (j=1, 2..n) represents the variables that need to be minimized/maximized simultaneously for a problem involving a set of ODE

$$\frac{dx}{dt} = F(x,u) \qquad (7)$$

t_f being the final time value, and n the total number of objective variables and . u the control parameter. First the single objective optimal control problem is solved independently optimizing each of the variables $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$. The minimization/maximization of $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ will lead to the values q_j^* . Then the optimization problem that will be solved is

$$\min\left(\sum_{j=1}^{n}\left(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^*\right)\right)^2 \qquad (8)$$
$$subject\ to\ \frac{dx}{dt} = F(x,u);$$

This will provide the values of u at various times. The first obtained control value of u is implemented and the rest are discarded. This procedure is repeated until the implemented and the first obtained control values are the same or if the Utopia point where ($\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) = q_j^*$ for all j) is obtained.

Pyomo (Hart et al, 2017)[29] is used for these calculations. Here, the differential equations are converted to a Nonlinear Program (NLP) using the orthogonal collocation method The NLP is solved using IPOPT (Wächter And Biegler, 2006)[30]and confirmed as a global solution with BARON (Tawarmalani, M. and N. V. Sahinidis 2005)[31].

The steps of the algorithm are as follows

1. Optimize $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ and obtain q_j^* at various time intervals t_i. The subscript i is the index for each time step.
2. Minimize $(\sum_{j=1}^{n}(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)-q_j^*))^2$ and get the control values for various times.
3. Implement the first obtained control values
4. Repeat steps 1 to 3 until there is an insignificant difference between the implemented and the first obtained value of the control variables or if the Utopia point is achieved. The Utopia point is when $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)=q_j^*$ for all j.

Sridhar (2024)[32] demonstrated that that the MNLMPC calculations would converge to the Utopia solution when the bifurcation analysis revealed the presence of limit and branch points . This was done by imposing the singularity condition on the co-state equation (Upreti, 2013)[33]. If the minimization of q_1 lead to the value q_1^* and the minimization of q_2 lead to the value q_2^* The MNLPMC calculations will minimize the function $(q_1-q_1^*)^2+(q_2-q_2^*)^2$. The multiobjective optimal control problem is

$$\min \quad (q_1-q_1^*)^2+(q_2-q_2^*)^2 \quad \textit{subject to } \frac{dx}{dt}=F(x,u) \tag{9}$$

Differentiating the objective function results in

$$\frac{d}{dx_i}((q_1-q_1^*)^2+(q_2-q_2^*)^2)=2(q_1-q_1^*)\frac{d}{dx_i}(q_1-q_1^*)+2(q_2-q_2^*)\frac{d}{dx_i}(q_2-q_2^*) \tag{10}$$

The Utopia point requires that both $(q_1-q_1^*)$ and $(q_2-q_2^*)$ are zero. Therefore

$$\frac{d}{dx_i}((q_1-q_1^*)^2+(q_2-q_2^*)^2)=0 \tag{11}$$

the optimal control co-state equation (Upreti; 2013)[45] is

$$\frac{d}{dt}(\lambda_i)=-\frac{d}{dx_i}((q_1-q_1^*)^2+(q_2-q_2^*)^2)-f_x\lambda_i; \quad \lambda_i(t_f)=0 \tag{12}$$

λ_i is the Lagrangian multiplier. t_f is the final time. The first term in this equation is 0 and hence

$$\frac{d}{dt}(\lambda_i)=-f_x\lambda_i;\lambda_i(t_f)=0 \tag{13}$$

At a limit or a branch point, for the set of ODE $\frac{dx}{dt} = f(x,u)$ f_x is singular. Hence there are two different vectors-values for $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) > 0$ and $\frac{d}{dt}(\lambda_i) < 0$. In between there is a vector $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) = 0$. This, coupled with the boundary condition $\lambda_i(t_f) = 0$ will lead to $[\lambda_i] = 0$ This makes the problem an unconstrained optimization problem, and the only solution is the Utopia solution.

5. RESULTS AND DISCUSSION

Bifurcation analysis for both models revealed the existence of branch points at $(J, Ps, Pi, V, u1)$ values of (50.0 0 0 25 0.6958) for model 1, (fig. 1) and (x, l, y, u, v, γ) values of (0.5 0.0 0.0 0.0 0.0 0.68) (fig. 2)

For the MNLMPC calculation in model 1 , $\sum_{t_{i=0}}^{t_i=t_f} Ps(t_i), \sum_{t_{i=0}}^{t_i=t_f} Pi(t_i)$ were minimized individually, and each led to a value of 0. The multiobjective optimal control problem will involve the minimization of $(\sum_{t_{i=0}}^{t_i=t_f} Ps(t_i) - 0)^2 + (\sum_{t_{i=0}}^{t_i=t_f} Pi(t_i) - 0)^2$ subject to the equations governing the model. This led to a value of zero (the Utopia solution). The MNLMPC control values obtained for u1 u2 were 0.0516 and 0.4931.

The various profiles for this MNLMPC calculation are shown in Figs. 3a,3b,3c and 3d. . The obtained control profile of u1 u2 exhibited noise (Fig. 3e). This issue was addressed using the Savitzky-Golay Filter. The smoothed version of this profile is shown in Fig. 3f . The MNLMPC calculations converged to the Utopia solution, validating the analysis by Sridhar (2024)[32], which demonstrated that the presence of a limit point/branch point enables the MNLMPC calculations to reach the optimal (Utopia) solution.

For the MNLMPC calculation in model 2, $\sum_{t_{i=0}}^{t_i=t_f} v(t_i), \sum_{t_{i=0}}^{t_i=t_f} y(t_i)$ were minimized individually, and each led to a value of 0. The multiobjective optimal control problem will involve the minimization of $(\sum_{t_{i=0}}^{t_i=t_f} v(t_i) - 0)^2 + (\sum_{t_{i=0}}^{t_i=t_f} y(t_i) - 0)^2$ subject to the equations governing the model. This led to a value of zero (the Utopia solution). The MNLMPC control values obtained for γ was 0.4015.

The various profiles for this MNLMPC calculation are shown in Figs. 4a,4b,4c 4d and 4e . The obtained control profile of γ exhibited noise (Fig. 4f). This issue was addressed using the Savitzky-Golay Filter. The smoothed version of this profile is shown in Fig. 4g. The MNLMPC calculations converged to the Utopia solution, validating the analysis by Sridhar (2024)[32], which demonstrated that the presence of a limit point/branch point enables the MNLMPC calculations to reach the optimal (Utopia) solution.

In both cases, the presence of a branch point enabled the MNLMPC calculations to reach the Utopia solution, validating the analysis by Sridhar (2024)[32].

6. CONCLUSIONS

Bifurcation analysis and Multiobjective nonlinear model predictive control calculations were performed on two dynamic agriculture models involving Jatropha curcas. The bifurcation analysis revealed the existence of a branch point in both problems. The branch points (which cause multiple steady-state solutions from a singular point) are very beneficial because they enable the multiobjective nonlinear model predictive control calculations to converge to the Utopia point (the best possible solution) in this model. A combination of bifurcation analysis and Multiobjective Nonlinear Model Predictive Control(MNLMPC) for agriculture problems is the main contribution of this paper.

DATA AVAILABILITY STATEMENT

All data used is presented in the paper

CONFLICT OF INTEREST

The author, Dr. Lakshmi N Sridhar has no conflict of interest.

ACKNOWLEDGEMENT

Dr. Sridhar thanks Dr. Carlos Ramirez and Dr. Suleiman for encouraging him to write single-author papers

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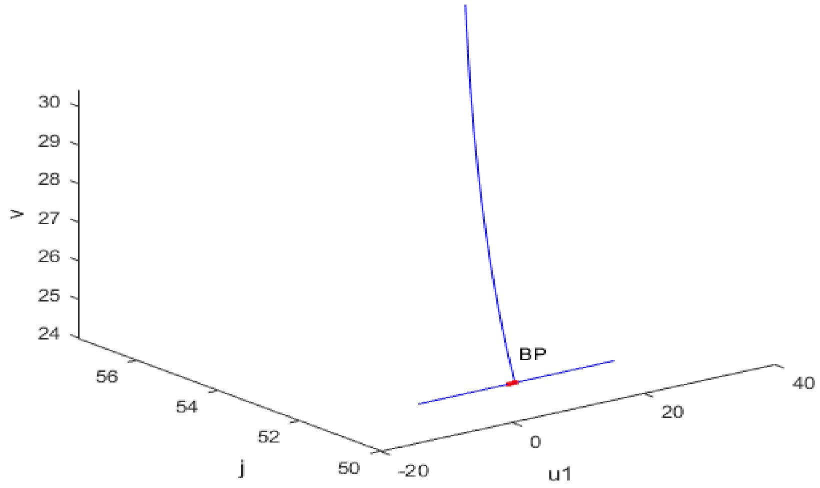

Fig. 1 Bifurcation analysis for model 1

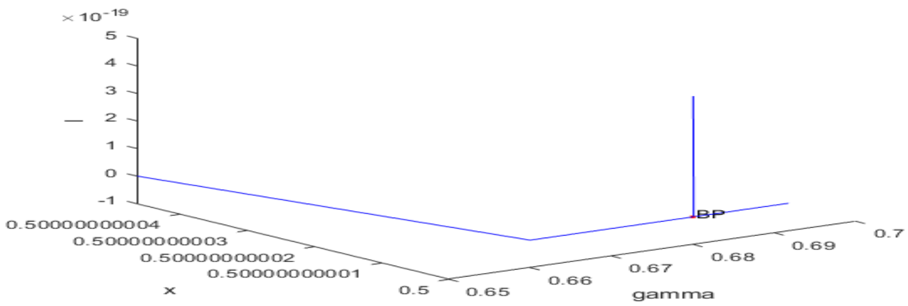

Fig. 2 Bifurcation analysis for model

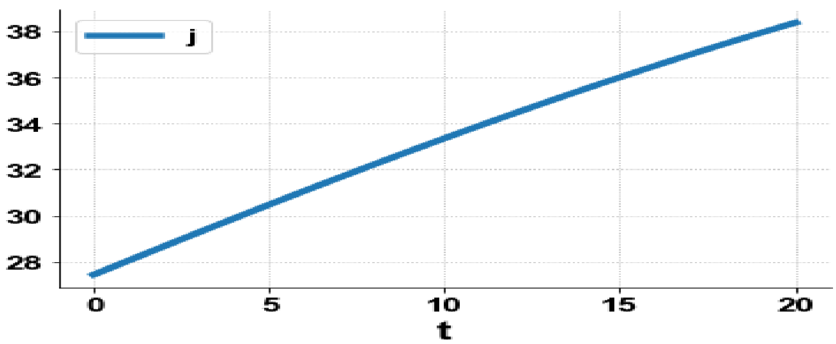

Fig. 3a MNLMPC model 1 j vs t

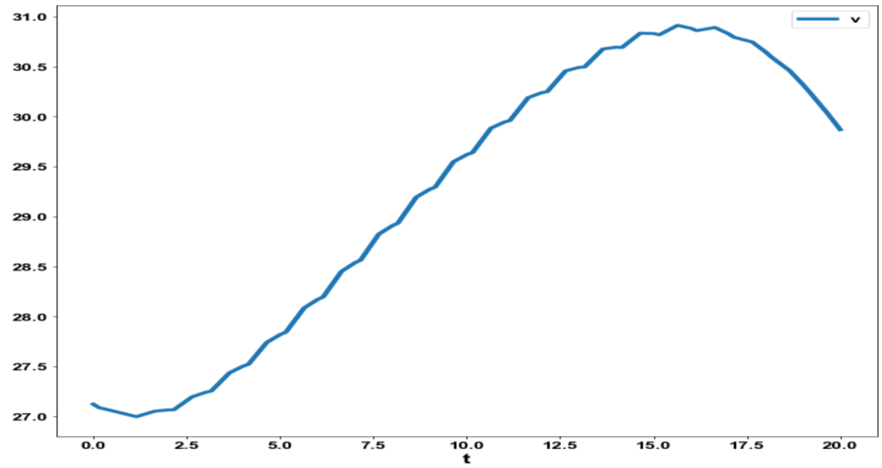

Fig. 3b MNLMPC model 1 v vs t

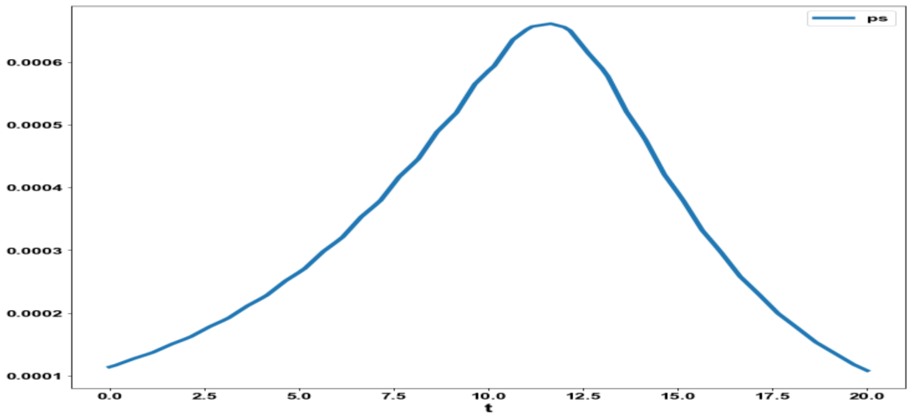

Fig. 3c MNLMPC model 1 Ps vs t

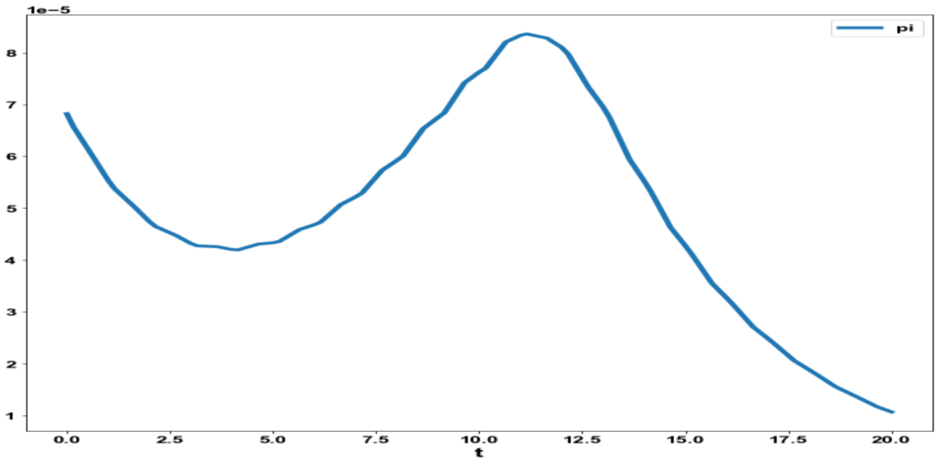

Fig. 3d MNLMPC model 1 Pi vs t

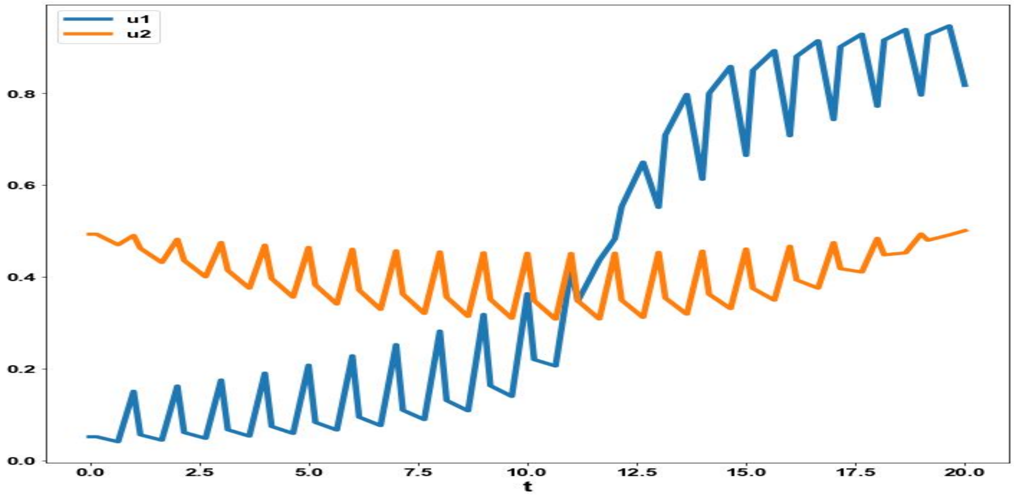

Fig. 3e MNLMPC model 1 u1 u2 vs t (noise exhibited)

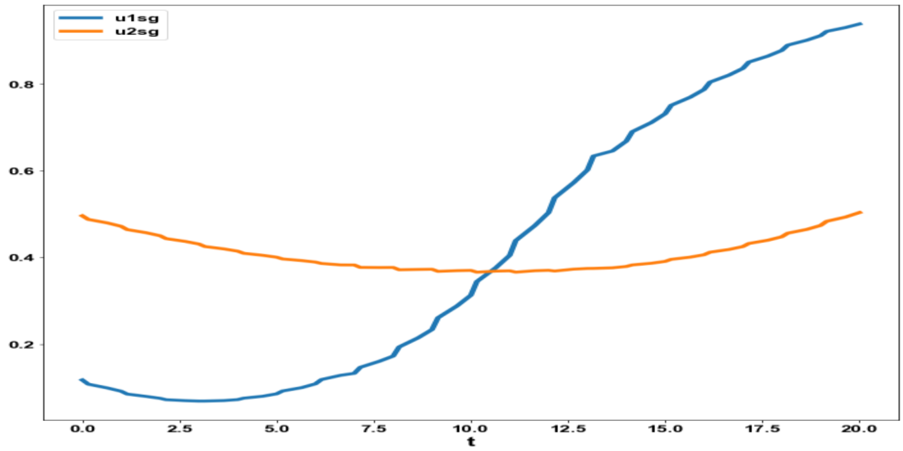

Fig. 3f MNLMPC model 1 u1 u2 (with Sazitzky Golay filter) vs t (noise eliminated)

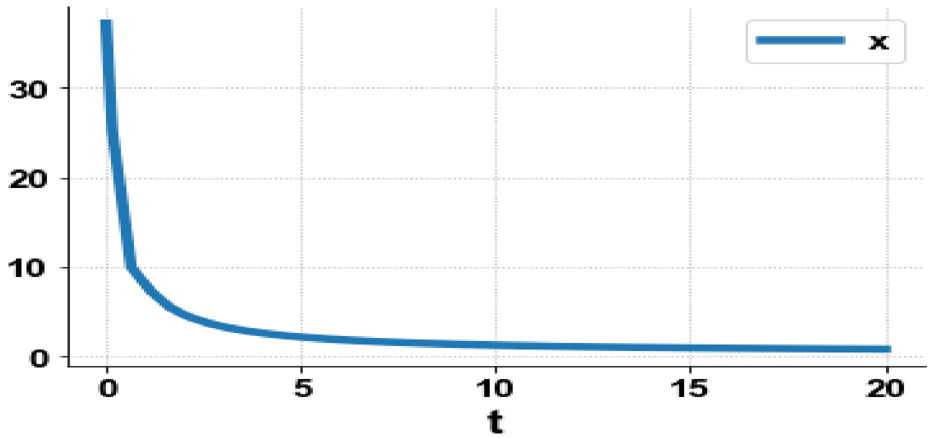

Fig. 4a MNLMPC model 2 x vs t

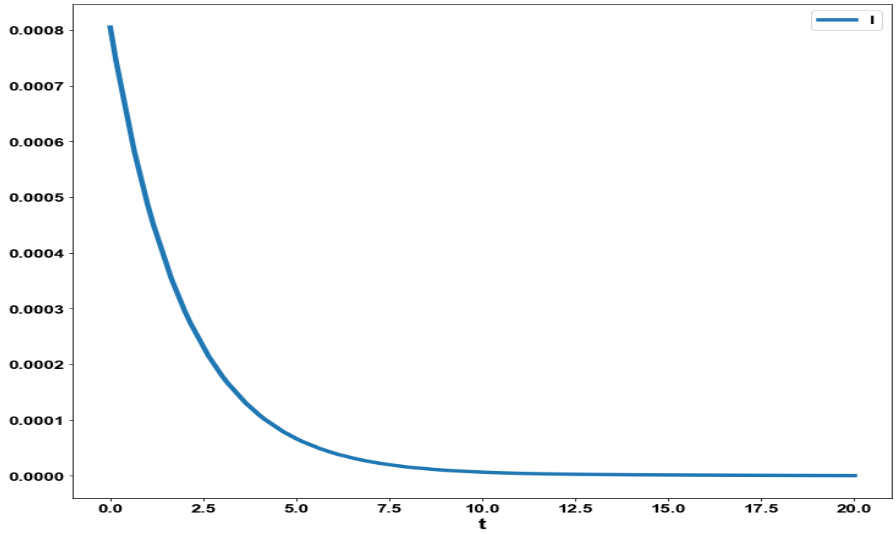

Fig. 4b MNLMPC model 2 l vs t

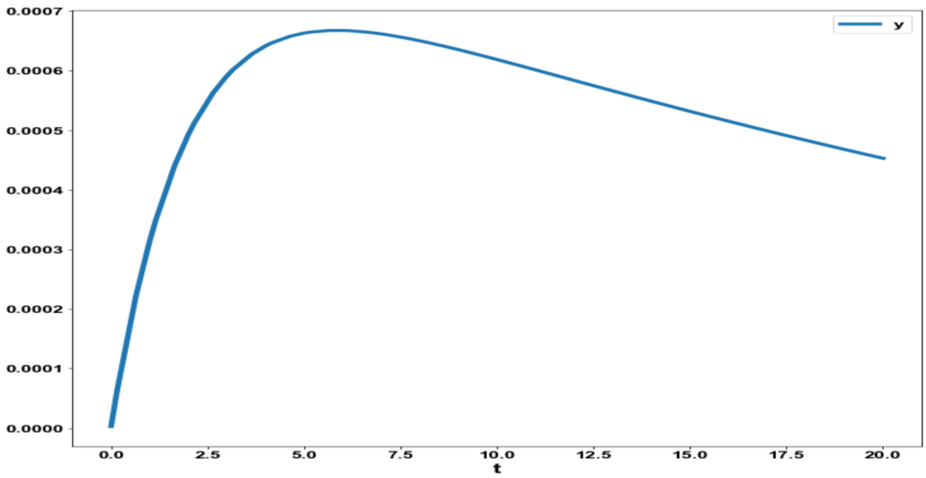

Fig. 4c MNLMPC model 2 y vs t

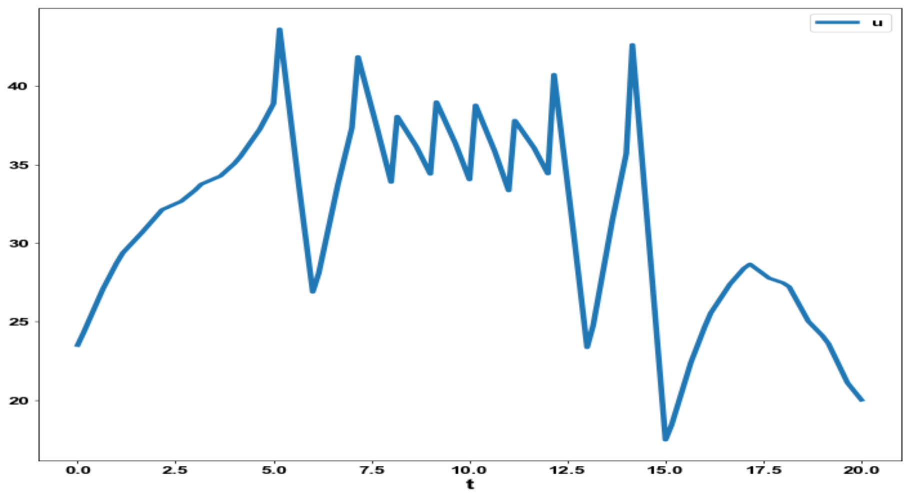

Fig. 4d MNLMPC model 2 u vs t

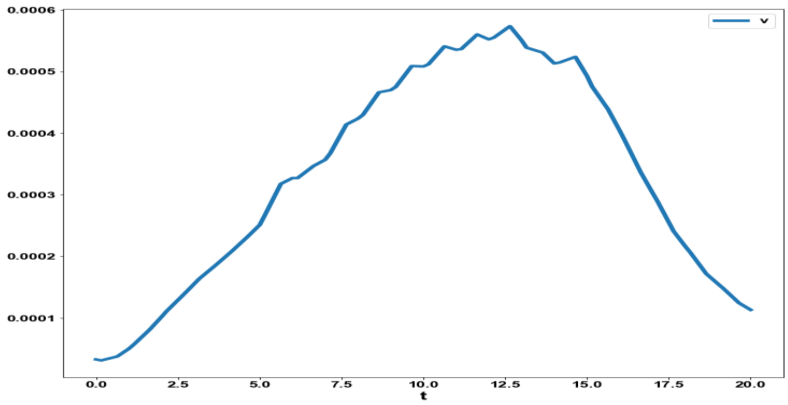

Fig. 4e MNLMPC model 2 v vs t

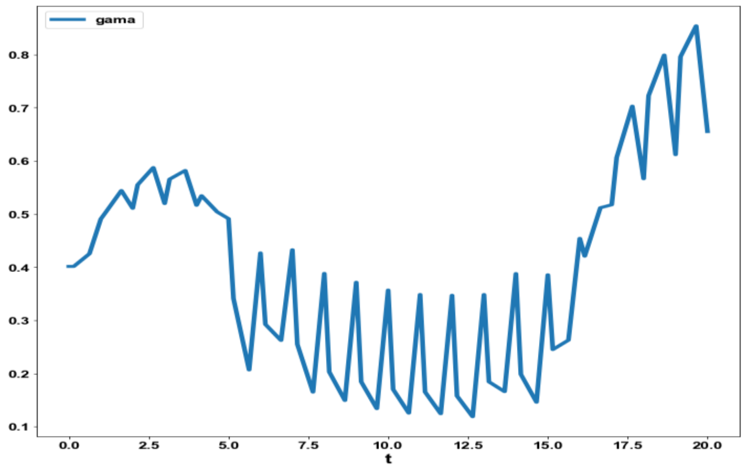

Fig. 4f MNLMPC model 2 gama vs t(noise exhibited)

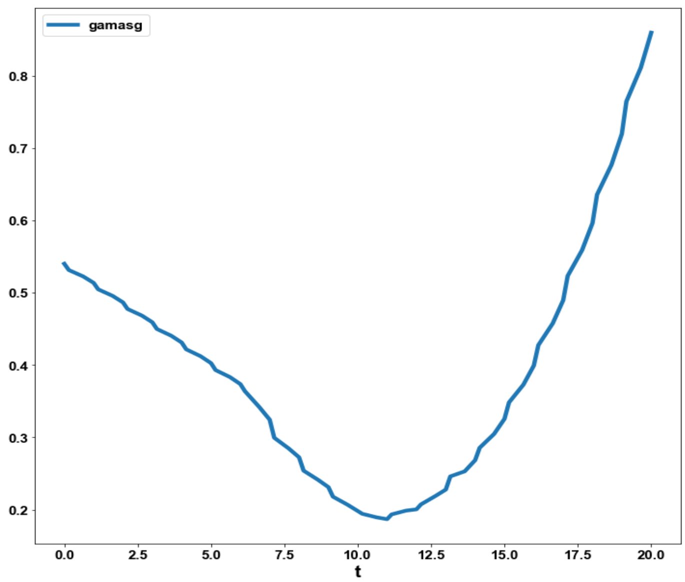

Fig. 4g MNLMPC model 2 gama (with Sazitzky Golay filter) vs t (noise eliminated)