ON APPROACH FOR ESTIMATION OF MAXIMAL CONTINUANCE OF DIFFUSION AND ION TYPE OF DOPING

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ABSTRACT

In this paper an analytical approach for estimation of maximal continuance of manufacturing of integrated circuit elements by dopant diffusion and ion implantation has been introduced. We analyzed influence of parameters of considered technological processes on the value of it's maximal continuance.

KEYWORDS

Manufacturing of integrated circuit elements; dopant diffusion; ion implantation; maximal continuance of technological processes; analytical approach for prognosis.

1. INTRODUCTION

One of the intensively solved problems for production of solid-state electronics devices is increasing of the integration rate of elements of integrated circuit (*p*-*n*- junctions; field-effect and bipolar transistors; ...), as well as increasing of their performance [1-8]. Different methods are using for manufacture of elements of integrated circuits. Some of them are ion and diffusion types of doping of required sections of electronic materials, epitaxial growth of multilayer structures, fusion of materials [9-17]. Main aim of the present paper is estimation of maximal continuance of ion and diffusion types of doping. The accompanying of the present paper is development of analytical approach for analysis of the considered continuance.

2. METHOD OF SOLUTION

In this section we determine spatio-temporal distributions of concentrations of infused and implanted dopants. To determine these distributions we calculate appropriate solutions of the second Fick's law [1,3,18,19]

$$\frac{\partial C(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_C \frac{\partial C(x,t)}{\partial x}\right] \tag{1}$$

Boundary and initial conditions for the equations are for finite source of dopant

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$$\left.\frac{\partial C(x,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial C(x,t)}{\partial x}\right|_{x=L}=0,\ C\,(x,0)=f\,(x); \tag{2a}$$

for infinite source of dopant

$$C\,(0,t)=C_0,\ \left.\frac{\partial C(x,t)}{\partial x}\right|_{x=L}=0,\ C\,(x>0,0)=0. \tag{2b}$$

The function $C(x,y,z,t)$ describes the spatio-temporal distribution of concentration of dopant; T is the temperature of annealing; D_C is the dopant diffusion coefficient. Value of dopant diffusion coefficient could be changed with changing materials of heterostructure, with changing temperature of materials (including annealing), with changing concentrations of dopant and radiation defects. We approximate dependences of dopant diffusion coefficient on parameters by the following relation with account results in Refs. [19-21]

$$D_C=D_L(x,T)\left[1+\xi\frac{C^{\gamma}(x,t)}{P^{\gamma}(x,T)}\right]\left[1+\varsigma_1\frac{V(x,t)}{V^*}+\varsigma_2\frac{V^2(x,t)}{(V^*)^2}\right]. \tag{3}$$

Here the function $D_L\,(x,T)$ describes the spatial (in heterostructure) and temperature (due to Arrhenius law) dependences of diffusion coefficient of dopant. The function $P\,(x,T)$ describes the limit of solubility of dopant. Parameter $\gamma\in[1,3]$ describes average quantity of charged defects interacted with atom of dopant [19]. The function $V\,(x,t)$ describes the spatio-temporal distribution of concentration of radiation vacancies with equilibrium distribution V^*. The considered concentrational dependence of dopant diffusion coefficient has been described in details in [19]. It should be noted, that using diffusion type of doping did not generation radiation defects. In this situation $\zeta_1=\zeta_2=0$. We determine spatio-temporal distributions of concentrations of radiation defects by solving the following system of equations [20,21]

$$\begin{cases}\dfrac{\partial I\,(x,t)}{\partial t}=\dfrac{\partial}{\partial x}\left[D_I(x,T)\dfrac{\partial I(x,t)}{\partial x}\right]-k_{I,I}(x,T)I^2(x,t)-k_{I,V}(x,T)I(x,t)V(x,t)\\[2ex]\dfrac{\partial V\,(x,t)}{\partial t}=\dfrac{\partial}{\partial x}\left[D_V(x,T)\dfrac{\partial V(x,t)}{\partial x}\right]-k_{V,V}(x,T)V^2(x,t)-k_{I,V}(x,T)I(x,t)V(x,t)\end{cases} \tag{4}$$

Boundary and initial conditions for these equations are

$$\left.\frac{\partial \rho(x,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial \rho(x,t)}{\partial x}\right|_{x=L}=0,\ \rho\,(x,0)=f_\rho\,(x). \tag{5}$$

Here $\rho=I,V$. The function $I\,(x,t)$ describes the spatio-temporal distribution of concentration of radiation interstitials with equilibrium distribution I^*; $D_\rho(x,T)$ are the diffusion coefficients of point radiation defects; terms $V^2(x,t)$ and $I^2(x,t)$ correspond to generation divacancies and diinterstitials; $k_{I,V}(x,T)$ is the parameter of recombination of point radiation defects; $k_{I,I}(x,T)$ and $k_{V,V}(x,T)$ are the parameters of generation of simplest complexes of point radiation defects.

Further we determine distributions in space and time of concentrations of divacancies $\Phi_V(x,t)$ and diinterstitials $\Phi_I(x,t)$ by solving the following system of equations [20,21]

$$\begin{cases}\frac{\partial \Phi_I(x,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi I}(x,T)\frac{\partial \Phi_I(x,t)}{\partial x}\right]+k_{I,I}(x,T)I^2(x,t)-k_I(x,T)I\,(x,t)\\ \frac{\partial \Phi_V(x,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi V}(x,T)\frac{\partial \Phi_V(x,t)}{\partial x}\right]+k_{V,V}(x,T)V^2(x,t)-k_V(x,T)\,V(x,t)\end{cases} \quad (6)$$

Boundary and initial conditions for these equations are

$$\left.\frac{\partial \Phi_\rho(x,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial \Phi_\rho(x,t)}{\partial x}\right|_{x=L}=0,\ \Phi_\rho(x,0)=f_{\Phi\rho}(x). \quad (7)$$

Here $D_{\Phi\rho}(x,T)$ are the diffusion coefficients of the above complexes of radiation defects; $k_I(x,T)$ and $k_V(x,T)$ are the parameters of decay of these complexes.

We calculate distributions of concentrations of point radiation defects in space and time by recently elaborated approach [22]. The approach based on transformation of approximations of diffusion coefficients in the following form: $D_\rho(x,T)=D_{0\rho}[1+\varepsilon_\rho\, g_\rho(x,\ T)]$, where $D_{0\rho}$ are the average values of diffusion coefficients, $0\le\varepsilon_\rho<1$, $|g_\rho(x,T)|\le1$. We also used analogous transformation of approximations of parameters of recombination of point defects and parameters of generation of their complexes: $k_{I,V}(x,T)=k_{0I,V}\,[1+\varepsilon_{I,V}g_{I,V}(x,T)]$, $k_{I,I}(x,T)=k_{0I,I}[1+\varepsilon_{I,I}\,g_{I,I}(x,T)]$ and $k_{V,V}(x,T)=k_{0V,V}[1+\varepsilon_{V,V}\,g_{V,V}(x,T)]$, where $k_{0\rho1,\rho2}$ are the their average values, $0\le\varepsilon_{I,V}<1$, $0\le\varepsilon_{I,I}<1$, $0\le\varepsilon_{V,V}<1$, $|\,g_{I,V}(x,T)|\le1$, $|\,g_{I,I}(x,\ T)|\le1$, $|g_{V,V}(x,T)|\le1$. Let us introduce the following dimensionless variables: $\tilde{I}(x,t)=I(x,t)/I^*$, $\tilde{V}(x,t)=V(x,t)/V^*$, $\omega=L^2k_{0I,V}\big/\sqrt{D_{0I}D_{0V}}$, $\chi = x/L_x$, $\Omega_\rho=L^2k_{0\rho,\rho}\big/\sqrt{D_{0I}D_{0V}}$, $\vartheta=\sqrt{D_{0I}D_{0V}}\,t/L^2$. The introduction leads to transformation of Eqs. (4) and conditions (5) to the following form

$$\begin{cases}\frac{\partial \tilde{I}(\chi,\vartheta)}{\partial \vartheta}=\frac{D_{0I}}{\sqrt{D_{0I}D_{0V}}}\frac{\partial}{\partial \chi}\left\{\left[1+\varepsilon_I g_I(\chi,T)\right]\frac{\partial \tilde{I}(\chi,\vartheta)}{\partial \chi}\right\}-\\ -\omega\left[1+\varepsilon_{I,V}g_{I,V}(\chi,T)\right]\tilde{I}(\chi,\vartheta)\tilde{V}(\chi,\vartheta)-\Omega_I\left[1+\varepsilon_{I,I}g_{I,I}(\chi,T)\right]\tilde{I}^2(\chi,\vartheta)\\ \frac{\partial \tilde{V}(\chi,\vartheta)}{\partial \vartheta}=\frac{D_{0V}}{\sqrt{D_{0I}D_{0V}}}\frac{\partial}{\partial \chi}\left\{\left[1+\varepsilon_V g_V(\chi,T)\right]\frac{\partial \tilde{V}(\chi,\vartheta)}{\partial \chi}\right\}-\\ -\omega\left[1+\varepsilon_{I,V}g_{I,V}(\chi,T)\right]\tilde{I}(\chi,\vartheta)\tilde{V}(\chi,\vartheta)-\Omega_V\left[1+\varepsilon_{V,V}g_{V,V}(\chi,T)\right]\tilde{V}^2(\chi,\vartheta)\end{cases} \quad (8)$$

$$\left.\frac{\partial \tilde{\rho}(\chi,\vartheta)}{\partial \chi}\right|_{\chi=0}=0,\ \left.\frac{\partial \tilde{\rho}(\chi,\vartheta)}{\partial \chi}\right|_{\chi=1}=0,\ \tilde{\rho}(\chi,\vartheta)=\frac{f_\rho(\chi,\vartheta)}{\rho^*}. \quad (9)$$

We determine solutions of Eqs. (8) with conditions (9) framework recently introduced approach [22], i.e. as the power series

$$\tilde{\rho}(\chi,\vartheta)=\sum_{i=0}^{\infty}\varepsilon_\rho^i\sum_{j=0}^{\infty}\omega^j\sum_{k=0}^{\infty}\Omega_\rho^k\tilde{\rho}_{ijk}(\chi,\vartheta). \quad (10)$$

Substitution of the series (10) into Eqs.(8) and conditions (9) gives us possibility to obtain equations for initial-order approximations of concentration of point defects $\tilde{\rho}_{000}(\chi,\vartheta)$ and corrections for them $\tilde{\rho}_{ijk}(\chi,\vartheta)$, $i\ge1$, $j\ge1$, $k\ge1$. The equations are presented in the Appendix.

Solutions of the equations could be obtained by standard Fourier approach [23,24]. The solutions are presented in the Appendix.

Now we calculate distributions of concentrations of simplest complexes of point radiation defects in space and time. To determine the distributions we transform approximations of diffusion coefficients in the following form: $D_{\Phi\rho}(x,T)=D_{0\Phi\rho}[1+\varepsilon_{\Phi\rho}\, g_{\Phi\rho}(x,T)]$, where $D_{0\Phi\rho}$ are the average values of diffusion coefficients. In this situation the Eqs.(6) could be written as

$$\begin{cases} \frac{\partial \Phi_I(x,t)}{\partial t} = D_{0\Phi I}\frac{\partial}{\partial x}\left\{\left[1+\varepsilon_{\Phi I}\, g_{\Phi I}(x,T)\right]\frac{\partial \Phi_I(x,t)}{\partial x}\right\} + k_{I,I}(x,T)\, I^2(x,t) - k_I(x,T)\, I\,(x,t) \\ \frac{\partial \Phi_V(x,t)}{\partial t} = D_{0\Phi V}\frac{\partial}{\partial x}\left\{\left[1+\varepsilon_{\Phi V}\, g_{\Phi V}(x,T)\right]\frac{\partial \Phi_V(x,t)}{\partial x}\right\} + k_{V,V}(x,T)\, V^2(x,t) - k_V(x,T)\, V\,(x,t) \end{cases}$$

Farther we determine solutions of above equations as the following power series

$$\Phi_\rho(x,t) = \sum_{i=0}^{\infty} \varepsilon_{\Phi\rho}^i \Phi_{\rho i}(x,t). \tag{11}$$

Now we used the series (11) into Eqs.(6) and appropriate boundary and initial conditions. The using gives the possibility to obtain equations for initial-order approximations of concentrations of complexes of defects $\Phi_{\rho 0}(x,t)$, corrections for them $\Phi_{\rho i}(x,t)$ (for them $i \geq 1$) and boundary and initial conditions for them. We remove equations and conditions to the Appendix. Solutions of the equations have been calculated by standard approaches [23,24] and presented in the Appendix.

Now we calculate distribution of concentration of dopant in space and time by using the approach, which was used for analysis of radiation defects. To use the approach we consider following transformation of approximation of dopant diffusion coefficient: $D_L(x,T)=D_{0L}[1+\varepsilon_L g_L(x,T)]$, where D_{0L} is the average value of dopant diffusion coefficient, $0 \leq \varepsilon_L < 1$, $|g_L(x,T)| \leq 1$. Farther we consider solution of Eq. (1) as the following series:

$$C(x,y,z,t) = \sum_{i=0}^{\infty} \varepsilon_L^i \sum_{j=1}^{\infty} \xi^j C_{ij}(x,t).$$

Using the relation into Eq. (1) and conditions (2) leads to obtaining equations for the functions $C_{ij}(x,t)$ ($i \geq 1$, $j \geq 1$), boundary and initial conditions for them. The equations are presented in the Appendix. Solutions of the equations have been calculated by standard approaches (see, for example, [23,24]). The solutions are presented in the Appendix. We analyzed distributions of concentrations of dopant and radiation defects in space and time analytically by using the second-order approximations on all parameters, which have been used in appropriate series. Usually the second-order approximations are enough good approximations to make qualitative analysis and to obtain quantitative results. All analytical results have been checked by numerical simulation.

Let us to use recently introduce criterion to estimate maximal value of continuance of technological process [25]. In the framework of the criterion let us approximate changing of considered concentrations in time by the following step-wise function (see Figs. 1-4)

$$\psi(x,t) = a_0 + a_1[1(t) - 1(t-\Theta)], \tag{12}$$

where $1(t)$ is the single step-wise function [26]. Not yet known parameters a_0, a_1 and Θ could have different values in different points of the considered material. Values of these parameters were determined were determined by minimization of the following the mean-squared error

$$U = \int_0^{t_N} \left[C(x,t) - \psi(x,t)\right]^2 dt \,, \tag{14}$$

where t_N is the observation time of transition process. Minimization of the mean- square error (14) gives a possibility to obtain the following relations for calculation of the considered parameters

$$\int_0^{t_N} C(x,t)\,d\,t = a_0 t_N + a_1\Theta \,, \tag{15a}$$

$$\int_0^{\Theta} C(x,t)\,d\,t = (a_0 + a_1)\Theta \,, \tag{15b}$$

$$C\,(x,\Theta)=a_0+0.5a_1. \tag{15c}$$

The criterion is optimal. However the approach did not leads to obtaining analytical relations for calculation of the considered maximal value of continuance of technological process. To obtain analytical relations for the considered relations it is attracted an interest asymptotically optimal criteria. To obtain transition to the criteria one shall consider the following limiting case $t_N \to \infty$. In this case one can obtain the following relations: $a_0 = C(x,\infty)$ and $a_1 = C(x,0)-C(x,\infty)$. Before consideration of the following limiting transition one shall the transform relation (15*a*) to the following form

$$\int_0^{t_N} \left[C(x,t) - a_0\right] d\,t = a_1\Theta \,.$$

Further obtaining of time of step-wise changing of approximation function (13) under condition of the limiting case $t_N \to \infty$ one can obtain the following criterion to estimate time scales, which known as rectangle with equal square

$$\Theta(x) = \frac{\int_0^{\infty} \left[C(x,t) - C(x,\infty)\right] d\,t}{C(x,0) - C(x,\infty)} \,. \tag{16}$$

Monotonous in time concentrations of dopant (see Figs. 1 and 2) could be approximated by the following functions

$$C(x,t) = \alpha\,[1-exp(-t/\tau)],\ C(x,t) = \beta \cdot exp(-t/\tau). \tag{17}$$

Substitution of the above relations into the relation (16) at fixed value of observation time of the diffusion process t_N gives a possibility to obtain the following relation for the considered time

$$\Theta = \tau\ [1-exp\,(-t_N/\tau)].$$

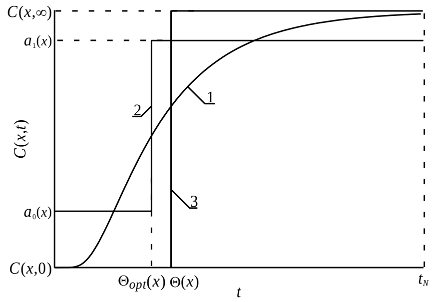

Fig. 1. Monotonic increasing of concentration of dopant (curve 1); optimal approximation of transition process, which was obtained by minimization of mean-squared error (14) (curve 2); asymptotically optimal approximation of transition process (curve 3)

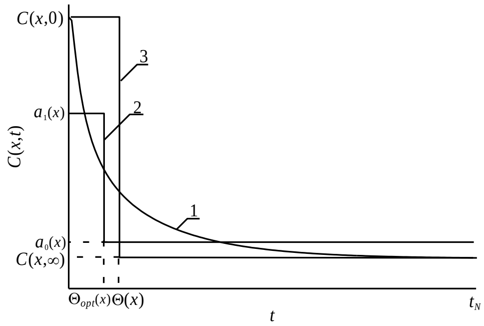

Fig. 2. Monotonic decreasing of concentration of dopant (curve 1); optimal approximation of transition process, which was obtained by minimization of mean-squared error (14) (curve 2); asymptotically optimal approximation of transition process (curve 3)

Consideration limiting case $t_N \to \infty$ leads to equality of single time scale of monotonous variation in time of dopant concentration and time scale, which was determined by relation (16). It should be noted, that relation (15*c*) at the limiting case $t_N \to \infty$ takes the form of another asymptotically optimal criterion. In the framework the second asymptotically optimal criterion maximal value maximal value of diffusion doping could be estimated as time of changed of the considered concentration in two times, i.e.

$$C\,(x,\Theta)=[C(x,0)+C(x,\infty)]/2. \quad (18)$$

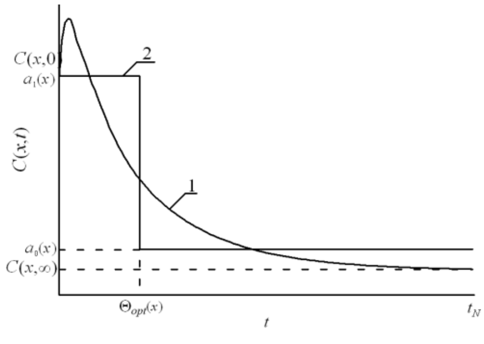

Fig. 3. Non-monotonic decreasing of concentration of dopant (curve 1); optimal approximation of transition process, which was obtained by minimization of mean-squared error (14) (curve 2); asymptotically optimal approximation of transition process (curve 3)

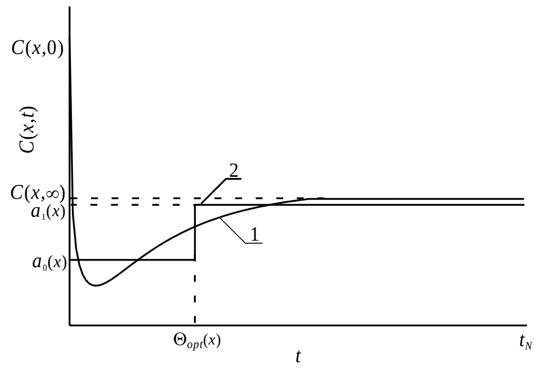

Fig. 4. Non-monotonic increasing of concentration of dopant (curve 1); optimal approximation of transition process, which was obtained by minimization of mean-squared error (14) (curve 2); asymptotically optimal approximation of transition process (curve 3)

However the first asymptotically optimal criterion (15) is nonlinear. Nonlinear criterion leads to obtaining smaller quantity of analytical relations for the considered maximal value of technological processes in comparison with criterion (16). In this situation we will use criterion (16) in future. However the criterion (16) has own disadvantage: the criterion could be used for monotonous in time concentrations of dopant. For non-monotonous in time concentrations of dopant the criterion (16) leads to underestimated values of the considered time. In this situation the considered time could takes negative values. It is attracted an interest maximal value of the considered time will be achieves, when initial distribution of concentration of infused dopant is presents near one boundary of the considered structure (i.e. $f(x)=\delta(x)$) and point of observation of this concentration is presented on other boundary of the considered structure (i.e. $x = L$), which should be doped. If the observation time on diffusion doping t_N is large in comparison with limiting time of technological process Θ, than transitions processes are absent at times $t > \Theta$.

3. DISCUSSION

In this section we analyzed limiting continuance of technological process for different profiles of diffusion coefficients without any variations in time (for example, annealing temperature is constant). Wide using have different multilayer structures. In this situation we will consider several normalized profiles of dopant diffusion coefficient $g(x)$, which are presented on Fig. 5. Analysis of limiting continuance of technological process shows, that in the case of infusion of dopant from finite source maximal variation of the considered continuance could be find in symmetrical structure with respect to it's middle (see Fig. 6). In the case of infusion of dopant from infinite source maximal variation of the considered continuance could be find in asymmetrical structure with respect to it's middle (see Fig. 7). Multilayer structures, which were presented on Figs. 6*a* and 7*a*, correspond to maximal increasing of the considered limiting continuance of technological processes (at fixed average value of dopant diffusion coefficient D_0). Multilayer structures, which were presented on Figs. 6*b* and 7*b*, correspond to maximal decreasing of the considered limiting continuance of technological processes (at fixed average value of dopant diffusion coefficient D_0).

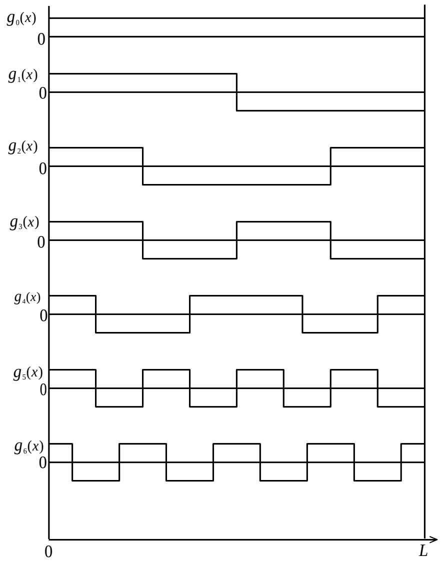

Fig. 5. Normalized profiles of dopant diffusion coefficients

We analyzed dependences of the considered limiting continuance on thicknesses of layers of multilayer structures. Variation of thicknesses of layers of multilayer structures not gives a possibility to find profiles of dopant diffusion coefficient, which correspond to larger influence on the considered continuance, in comparison with profiles, which were presented on Figs. 6 and 7. Increasing of quantity of layers of the considered multilayer structures leads to decreasing of influence of variation of dopant diffusion coefficient on the limiting continuance of technological process. Figs. 8 show dependences of the considered continuance on the value of parameter ε for profiles of dopant diffusion coefficient, which were presented on Figs. 6 and 7. These figures show, that the considered continuance could be decreased on several percents and increased on several orders in comparison with continuance Θ_0 for averaged value of dopant diffusion coefficient D_0. The continuance Θ_0 for averaged value of dopant diffusion coefficient D_0 is equal to $\Theta_0=L^2/6D_0$ for finite source of dopant and $\Theta_0=L^2/2D_0$ for infinite source of dopant.

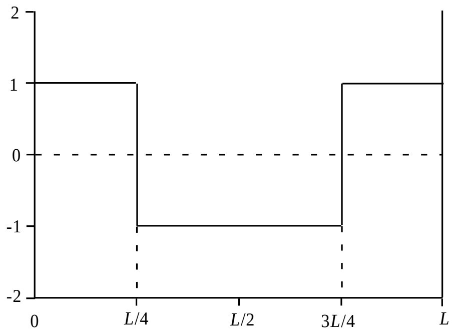

Fig. 6*a*. Normalized profiles of dopant diffusion coefficient, which correspond to maximal increasing of the limiting continuance of diffusion doping from finite source of dopant

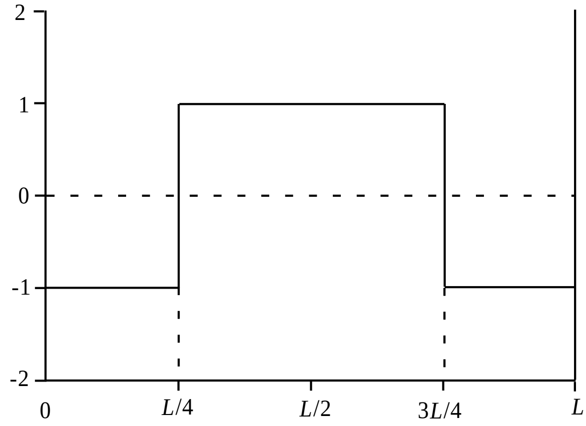

Fig. 6*b*. Normalized profiles of dopant diffusion coefficient, which correspond to maximal decreasing of the limiting continuance of diffusion doping from finite source of dopant

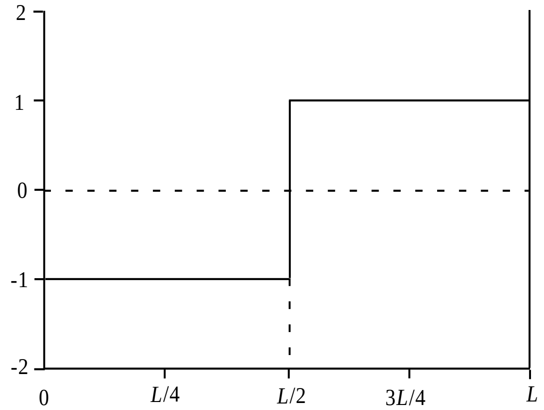

Fig. 7*a*. Normalized profiles of dopant diffusion coefficient, which correspond to maximal increasing of the limiting continuance of diffusion doping from infinite source of dopant

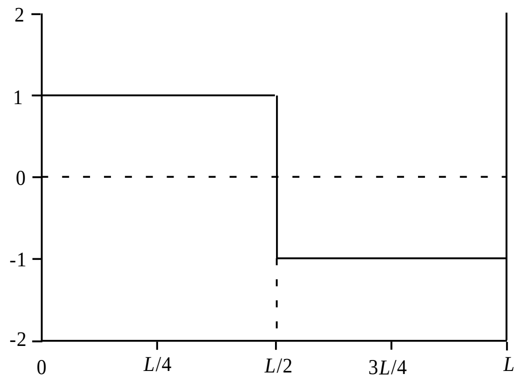

Fig. 7*b*. Normalized profiles of dopant diffusion coefficient, which correspond to maximal decreasing of the limiting continuance of diffusion doping from infinite source of dopant

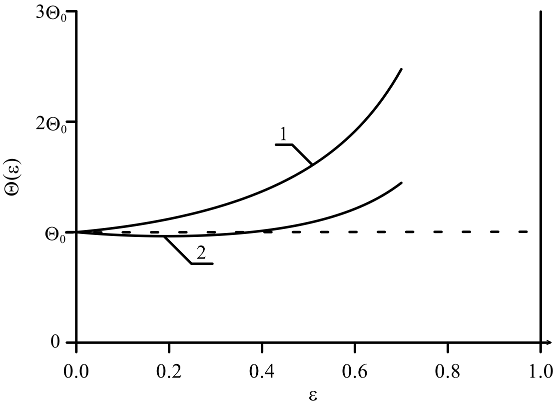

Fig. 8*a*. Dependences of limiting continuance of diffusion doping for finite source of dopant on value of parameter ε. Curve 1 corresponds to profile of dopant diffusion coefficient with decreased limiting continuance of technological process (see Fig. 6*b*). Curve 2 corresponds to profile of dopant diffusion coefficient with increased limiting continuance of technological process (see Fig. 6*a*)

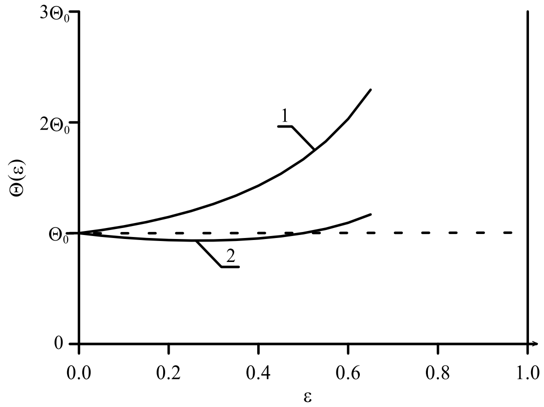

Fig. 8*b*. Dependences of limiting continuance of diffusion doping for infinite source of dopant on value of parameter ε. Curve 1 corresponds to profile of dopant diffusion coefficient with decreased limiting continuance of technological process (see Fig. 7*b*). Curve 2 corresponds to profile of dopant diffusion coefficient with increased limiting continuance of technological process (see Fig. 7*a*)

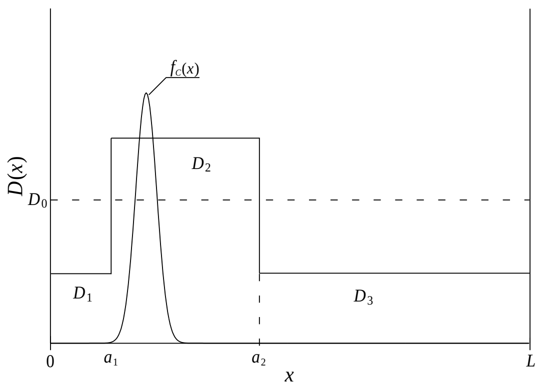

Fig. 9. Profile of dopant diffusion coefficient, which corresponds to maximal increasing of limiting continuance of ion doping. Profile of dopant diffusion coefficient, which corresponds to maximal decreasing of limiting continuance of ion doping, has the same difference with the above profile as for profiles 6*b* и 6*a*

Now let us consider influence of temporal variations of dopant diffusion coefficient on value of limiting continuance of technological process in homogenous material. The considered situation could be consider, for example, for nonstationary annealing of dopant and/or radiation defects, which are presents in homogenous material. In this case (as for multilayer structure) increasing of the considered limiting continuance is essentially smaller, than decreasing one at fixed value of averaged diffusion coefficient. The same conclusion could be obtained during analysis of joint changing of dopant diffusion coefficient in space and time.

Analogous conclusions about influence of variations of dopant diffusion coefficient on limiting continuance of technological process could be obtained for ion type of doping. At the same time one can find changing of thicknesses of layers of multilayer structures (see Fig. 9). This changing taking into account presents of maximal value of concentration of dopant in depth of the multilayer structure, but not on it's external boundary as for diffusion doping. Also qualitatively similar influence of spatial and temporal variations of dopant diffusion coefficients of radiation defects and accounted other parameters (parameters of recombination of point radiation defects; parameters of generation of complexes of point defects; parameters of decay of complexes of radiation defects) on limiting value of continuance of annealing time.

4. CONCLUSIONS

In this paper we introduce an analytical approach to estimate limiting value of continuance of technological process during doping (doping by diffusion; ion doping) of materials to manufacture elements of integrated circuits. We analyzed influence of parameters on value of the considered limiting continuance.

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APPENDIX

Equations for functions $\tilde{I}_{ijk}(\chi,\vartheta)$ andи $\tilde{V}_{ijk}(\chi,\vartheta)$, $i\geq 0$, $j\geq 0$, $k\geq 0$ and conditions for them could be written as

$$\frac{\partial \tilde{I}_{000}(\chi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0I}}{D_{0V}}}\frac{\partial^2 \tilde{I}_{000}(\chi,\vartheta)}{\partial \chi^2},\ \frac{\partial \tilde{V}_{000}(\chi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0V}}{D_{0I}}}\frac{\partial^2 \tilde{V}_{000}(\chi,\vartheta)}{\partial \chi^2};$$

$$\begin{cases}\frac{\partial \tilde{I}_{i00}(\chi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0I}}{D_{0V}}}\frac{\partial^2 \tilde{I}_{i00}(\chi,\vartheta)}{\partial \chi^2}+\sqrt{\frac{D_{0I}}{D_{0V}}}\frac{\partial}{\partial \chi}\left[g_I(\chi,T)\frac{\partial \tilde{I}_{i-100}(\chi,\vartheta)}{\partial \chi}\right]\\ \frac{\partial \tilde{V}_{i00}(\chi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0V}}{D_{0I}}}\frac{\partial^2 \tilde{V}_{i00}(\chi,\vartheta)}{\partial \chi^2}+\sqrt{\frac{D_{0V}}{D_{0I}}}\frac{\partial}{\partial \chi}\left[g_V(\chi,T)\frac{\partial \tilde{V}_{i-100}(\chi,\vartheta)}{\partial \chi}\right]\end{cases},\ i\geq 1;$$

$$\begin{cases}\frac{\partial \tilde{I}_{010}(\chi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0I}}{D_{0V}}}\frac{\partial^2 \tilde{I}_{010}(\chi,\vartheta)}{\partial \chi^2}-\left[1+\varepsilon_{I,V}\,g_{I,V}(\chi,T)\right]\tilde{I}_{000}(\chi,\vartheta)\,\tilde{V}_{000}(\chi,\vartheta)\\ \frac{\partial \tilde{V}_{010}(\chi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0V}}{D_{0I}}}\frac{\partial^2 \tilde{V}_{010}(\chi,\vartheta)}{\partial \chi^2}-\left[1+\varepsilon_{I,V}\,g_{I,V}(\chi,T)\right]\tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta)\end{cases};$$

$$\begin{cases}\frac{\partial \tilde{I}_{020}(\chi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0I}}{D_{0V}}}\frac{\partial^2 \tilde{I}_{020}(\chi,\vartheta)}{\partial \chi^2}-\\ -\left[1+\varepsilon_{I,V}\,g_{I,V}(\chi,T)\right]\left[\tilde{I}_{010}(\chi,\vartheta)\,\tilde{V}_{000}(\chi,\vartheta)+\tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{010}(\chi,\vartheta)\right]\\ \frac{\partial \tilde{V}_{020}(\chi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0I}}{D_{0V}}}\frac{\partial^2 \tilde{V}_{020}(\chi,\vartheta)}{\partial \chi^2}-\\ -\left[1+\varepsilon_{I,V}\,g_{I,V}(\chi,T)\right]\left[\tilde{I}_{010}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta)+\tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{010}(\chi,\vartheta)\right]\end{cases};$$

$$\begin{cases} \frac{\partial \tilde{I}_{001}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{I}_{001}(\chi,\vartheta)}{\partial \chi^2} - [1+\varepsilon_{I,I} g_{I,I}(\chi,T)] \tilde{I}_{000}^2(\chi,\vartheta) \\ \frac{\partial \tilde{V}_{001}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{V}_{001}(\chi,\vartheta)}{\partial \chi^2} - [1+\varepsilon_{I,I} g_{V,V}(\chi,T)] \tilde{V}_{000}^2(\chi,\vartheta) \end{cases};$$

$$\begin{cases} \frac{\partial \tilde{I}_{110}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{I}_{110}(\chi,\vartheta)}{\partial \chi^2} + \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial}{\partial \chi}\left[g_I(\chi,T) \frac{\partial \tilde{I}_{010}(\chi,\vartheta)}{\partial \chi}\right] - \\ -[1+\varepsilon_{I,I} g_{I,I}(\chi,T)][\tilde{I}_{100}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta) + \tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{100}(\chi,\vartheta)] \\ \frac{\partial \tilde{V}_{110}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{V}_{110}(\chi\vartheta)}{\partial \chi^2} + \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial}{\partial \chi}\left[g_V(\chi,T) \frac{\partial \tilde{V}_{010}(\chi,\vartheta)}{\partial \chi}\right] - \\ -[1+\varepsilon_{V,V} g_{V,V}(\chi,T)][\tilde{I}_{0100}(\chi,\vartheta)\tilde{V}_{100}(\chi,\vartheta) + \tilde{I}_{100}(\chi,\vartheta)\tilde{V}_{100}(\chi,\vartheta)] \end{cases};$$

$$\begin{cases} \frac{\partial \tilde{I}_{002}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{I}_{002}(\chi,\vartheta)}{\partial \chi^2} - [1+\varepsilon_{I,I} g_{I,I}(\chi,T)]\tilde{I}_{001}(\chi,\vartheta)\tilde{I}_{000}(\chi,\vartheta) \\ \frac{\partial \tilde{V}_{002}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{V}_{002}(\chi,\vartheta)}{\partial \chi^2} - [1+\varepsilon_{V,V} g_{V,V}(\chi,T)]\tilde{V}_{001}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta) \end{cases};$$

$$\begin{cases} \frac{\partial \tilde{I}_{101}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{I}_{101}(\chi,\vartheta)}{\partial \chi^2} + \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial}{\partial \chi}\left[g_I(\chi,T) \frac{\partial \tilde{I}_{001}(\chi,\vartheta)}{\partial \chi}\right] - \\ \qquad -[1+\varepsilon_I g_I(\chi,T)]\tilde{I}_{100}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta) \\ \frac{\partial \tilde{V}_{101}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{V}_{101}(\chi,\vartheta)}{\partial \chi^2} + \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial}{\partial \chi}\left[g_V(\chi,T) \frac{\partial \tilde{V}_{001}(\chi,\vartheta)}{\partial \chi}\right] - \\ \qquad -[1+\varepsilon_V g_V(\chi,T)]\tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{100}(\chi,\vartheta) \end{cases};$$

$$\begin{cases} \frac{\partial \tilde{I}_{011}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{I}_{011}(\chi,\vartheta)}{\partial \chi^2} - [1+\varepsilon_{I,I} g_{I,I}(\chi,T)]\tilde{I}_{000}(\chi,\vartheta)\tilde{I}_{010}(\chi,\vartheta) - \\ -[1+\varepsilon_{I,V} g_{I,V}(\chi,T)]\tilde{I}_{001}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta) \\ \frac{\partial \tilde{V}_{011}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{V}_{011}(\chi,\vartheta)}{\partial \chi^2} - [1+\varepsilon_{V,V} g_{V,V}(\chi,T)]\tilde{V}_{000}(\chi,\vartheta)\tilde{V}_{010}(\chi,\vartheta) - \\ -[1+\varepsilon_{I,V} g_{I,V}(\chi,t)]\tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{001}(\chi,\vartheta) \end{cases};$$

$$\left.\frac{\partial \tilde{\rho}_{ijk}(\chi,\vartheta)}{\partial \chi}\right|_{x=0} = 0,\ \left.\frac{\partial \tilde{\rho}_{ijk}(\chi,\vartheta)}{\partial \chi}\right|_{x=1} = 0\ (i \ge 0, j \ge 0, k \ge 0);\ \tilde{\rho}_{000}(\chi,0) = \frac{f_\rho(\chi)}{\rho^*},$$

$$\tilde{\rho}_{ijk}(\chi,0) = 0\ (i \ge 1, j \ge 1, k \ge 1).$$

Solutions of the above equations with appropriate boundary and initial conditions could be written as

$$\tilde{\rho}_{000}(\chi,\vartheta) = \frac{F_{0\rho}}{L} + \frac{2}{L}\sum_{n=1}^{\infty} F_{n\rho} c(\chi)\, e_{n\rho}(\vartheta),$$

where $F_{n\rho} = \frac{1}{\rho^*}\int_0^1 \cos(\pi n u) f_{n\rho}(u) du$, $e_{nI}(\vartheta) = \exp\left(-\pi^2 n^2 \vartheta \sqrt{D_{0V}/D_{0I}}\right)$, $c_n(\chi) = cos(\pi\ n\ \chi)$, $e_{nV}(\vartheta) = \exp\left(-\pi^2 n^2 \vartheta \sqrt{D_{0I}/D_{0V}}\right)$;

$$\begin{cases}\tilde{I}_{i00}(\chi,\vartheta)=-2\pi\sqrt{\frac{D_{0I}}{D_{0V}}}\sum_{n=1}^{\infty}n\,c_n(\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\int_0^1 s_n(u)g_I(u,T)\frac{\partial\tilde{I}_{i-100}(u,\tau)}{\partial u}d\,u\,d\,\tau\\ \tilde{V}_{i00}(\chi,\vartheta)=-2\pi\sqrt{\frac{D_{0V}}{D_{0I}}}\sum_{n=1}^{\infty}n\,c_n(\chi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\int_0^1 s_n(u)g_V(u,T)\frac{\partial\tilde{V}_{i-100}(u,\tau)}{\partial u}d\,u\,d\,\tau\end{cases},\ i\geq 1,$$

where $s_n(\chi) = sin\,(\pi\,n\,\chi)$;

$$\tilde{\rho}_{010}(\chi,\eta,\phi,\vartheta)=-2\sum_{n=1}^{\infty}c_n(\chi)c_n(\eta)c_n(\phi)e_{n\rho}(\vartheta)\int_0^{\vartheta}e_{n\rho}(-\tau)\int_0^1 c_n(u)\int_0^1 c_n(v)\int_0^1 c_n(w)\times$$
$$\times\left[1+\varepsilon_{I,V}g_{I,V}(u,v,w,T)\right]\tilde{I}_{000}(u,v,w,\tau)\,\tilde{V}_{000}(u,v,w,\tau)\,d\,w\,d\,v\,d\,u\,d\,\tau\,;$$

$$\tilde{\rho}_{020}(\chi,\vartheta)=-2\sqrt{\frac{D_{0I}}{D_{0V}}}\sum_{n=1}^{\infty}c_n(\chi)e_{n\rho}(\vartheta)\int_0^{\vartheta}e_{n\rho}(-\tau)\int_0^1 c_n(u)\left[1+\varepsilon_{I,V}g_{I,V}(u,T)\right]\times$$
$$\times\left[\tilde{I}_{010}(u,\tau)\tilde{V}_{000}(u,\tau)+\tilde{I}_{000}(u,\tau)\tilde{V}_{010}(u,\tau)\right]d\,u\,d\,\tau\,;$$

$$\tilde{\rho}_{001}(\chi,\vartheta)=-2\sum_{n=1}^{\infty}c_n(\chi)e_{n\rho}(\vartheta)\int_0^{\vartheta}e_{n\rho}(-\tau)\int_0^1 c_n(u)\left[1+\varepsilon_{\rho,\rho}g_{\rho,\rho}(u,T)\right]\tilde{\rho}_{000}^2(u,\tau)\,d\,u\,d\,\tau\,;$$

$$\tilde{\rho}_{002}(\chi,\vartheta)=-2\sum_{n=1}^{\infty}c_n(\chi)\,e_{n\rho}(\vartheta)\int_0^{\vartheta}e_{n\rho}(-\tau)\int_0^1 c_n(u)\left[1+\varepsilon_{\rho,\rho}g_{\rho,\rho}(u,T)\right]\tilde{\rho}_{001}(u,\tau)\tilde{\rho}_{000}(u,\tau)\,d\,u\,d\,\tau\,;$$

$$\begin{cases}\tilde{I}_{110}(\chi,\vartheta)=-2\pi\sqrt{\frac{D_{0I}}{D_{0V}}}\sum_{n=1}^{\infty}n\,c_n(\chi)\,e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\int_0^1 s_n(u)g_I(u,T)\frac{\partial\tilde{I}_{i-100}(u,\tau)}{\partial u}d\,u\,d\,\tau-\\ -2\sum_{n=1}^{\infty}\int_0^{\vartheta}e_{nI}(-\tau)\int_0^1 c_n(u)\left[1+\varepsilon_{I,V}g_{I,V}(u,T)\right]\left[\tilde{I}_{100}(u,\tau)\tilde{V}_{000}(u,\tau)+\tilde{I}_{000}(u,\tau)\tilde{V}_{100}(u,\tau)\right]d\,u\,d\,\tau\times\\ \times e_{nI}(\vartheta)\,c_n(\chi)\\ \tilde{V}_{110}(\chi,\vartheta)=-2\pi\sqrt{\frac{D_{0V}}{D_{0I}}}\sum_{n=1}^{\infty}n\,c_n(\chi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1 s_n(u)g_V(u,T)\frac{\partial\tilde{V}_{i-100}(u,\tau)}{\partial u}d\,u\,d\,\tau-\\ -2\sum_{n=1}^{\infty}\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1 c_n(u)\left[1+\varepsilon_{I,V}g_{I,V}(u,T)\right]\left[\tilde{I}_{000}(u,\tau)\tilde{V}_{100}(u,\tau)+\tilde{I}_{100}(u,\tau)\tilde{V}_{000}(u,\tau)\right]d\,u\,d\,\tau\times\\ \times c_n(\chi)\,e_{nV}(\vartheta)\end{cases};$$

$$\begin{cases}\tilde{I}_{101}(\chi,\vartheta)=-2\pi\sqrt{\frac{D_{0I}}{D_{0V}}}\sum_{n=1}^{\infty}n\,c_n(\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\int_0^1 s_n(u)g_I(u,T)\frac{\partial\tilde{I}_{001}(u,\tau)}{\partial u}d\,u\,d\,\tau-\\ -2\sum_{n=1}^{\infty}c_n(\chi)\,e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\int_0^1 c_n(u)\left[1+\varepsilon_{I,V}g_{I,V}(u,T)\right]\tilde{I}_{100}(u,\tau)\tilde{V}_{000}(u,\tau)d\,u\,d\,\tau\\ \tilde{V}_{101}(\chi,\vartheta)=-2\pi\sqrt{\frac{D_{0V}}{D_{0I}}}\sum_{n=1}^{\infty}n\,c_n(\chi)\,e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1 s_n(u)g_V(u,T)\frac{\partial\tilde{V}_{001}(u,\tau)}{\partial u}d\,u\,d\,\tau-\\ -2\sum_{n=1}^{\infty}c_n(\chi)\,e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1 c_n(u)\left[1+\varepsilon_{I,V}g_{I,V}(u,T)\right]\tilde{I}_{000}(u,\tau)\tilde{V}_{100}(u,\tau)d\,u\,d\,\tau\end{cases};$$

$$\begin{cases}\tilde{I}_{011}(\chi,\vartheta)=-2\sum_{n=1}^{\infty}c_n(\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\int_0^1 c_n(u)\left\{\left[1+\varepsilon_{I,I}g_{I,I}(u,T)\right]\tilde{I}_{000}(u,\tau)\tilde{I}_{010}(u,\tau)+\right.\\ \left.+\left[1+\varepsilon_{I,V}g_{I,V}(u,T)\right]\tilde{I}_{001}(u,\tau)\tilde{V}_{000}(u,\tau)\right\}d\,u\,d\,\tau\\ \tilde{V}_{011}(\chi,\vartheta)=-2\sum_{n=1}^{\infty}c_n(\chi)\,e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1 c_n(u)\left\{\left[1+\varepsilon_{V,V}g_{V,V}(u,T)\right]\tilde{V}_{000}(u,\tau)\tilde{V}_{010}(u,\tau)+\right.\\ \left.+\left[1+\varepsilon_{I,V}g_{I,V}(u,T)\right]\tilde{I}_{000}(u,\tau)\tilde{V}_{001}(u,\tau)\right\}d\,u\,d\,\tau\end{cases}$$

Equations for functions $\Phi_{\rho i}(x,t)$, $i \geq 0$, boundary and initial conditions takes the form

$$\begin{cases} \frac{\partial \Phi_{I0}(x,t)}{\partial t} = D_{0\Phi I} \frac{\partial^2 \Phi_{I0}(x,t)}{\partial x^2} + k_{I,I}(x,T)\, I^2(x,t) - k_I(x,T) I(x,t) \\ \frac{\partial \Phi_{V0}(x,t)}{\partial t} = D_{0\Phi V} \frac{\partial^2 \Phi_{V0}(x,t)}{\partial x^2} + k_{V,V}(x,T)\, V^2(x,t) - k_V(x,T) V(x,t) \end{cases};$$

$$\begin{cases} \frac{\partial \Phi_{Ii}(x,t)}{\partial t} = D_{0\Phi I} \frac{\partial^2 \Phi_{Ii}(x,t)}{\partial x^2} + D_{0\Phi I} \frac{\partial}{\partial x}\left[g_{\Phi I}(x,T) \frac{\partial \Phi_{Ii-1}(x,t)}{\partial x} \right] \\ \frac{\partial \Phi_{Vi}(x,t)}{\partial t} = D_{0\Phi V} \frac{\partial^2 \Phi_{Vi}(x,t)}{\partial x^2} + D_{0\Phi V} \frac{\partial}{\partial x}\left[g_{\Phi V}(x,T) \frac{\partial \Phi_{Vi-1}(x,t)}{\partial x} \right] \end{cases},\ i\ge 1;$$

$$\left.\frac{\partial \Phi_{\rho i}(x,t)}{\partial x}\right|_{x=0} = 0,\ \left.\frac{\partial \Phi_{\rho i}(x,t)}{\partial x}\right|_{x=L} = 0,\ i\ge 0;\ \Phi_{\rho 0}(x,0)=f_{\Phi\rho}(x),\ \Phi_{\rho i}(x,0)=0,\ i\ge 1.$$

Solutions of the above equations could be written as

$$\Phi_{\rho 0}(x,t) = \frac{F_{0\Phi_\rho}}{L} + \frac{2}{L}\sum_{n=1}^{\infty} F_{n\Phi_\rho} c_n(x) e_{n\Phi_\rho}(t) + \frac{2}{L}\sum_{n=1}^{\infty} n\, c_n(x) e_{\Phi_\rho n}(t) \times$$
$$\times \int_0^t e_{\Phi_\rho n}(-\tau) \int_0^L c_n(u)\left[k_{I,I}(u,T) I^2(u,\tau) - k_I(u,T) I(u,\tau)\right] d u\, d\tau ,$$

where $e_{n\Phi_\rho}(t) = \exp\left(-\pi^2 n^2 D_{0\Phi_\rho} t / L^2\right)$, $F_{n\Phi_\rho} = \int_0^{L_x} c_n(u) f_{\Phi_\rho}(u)\, d u$;

$$\Phi_{\rho i}(x,y,z,t) = -\frac{2\pi}{L^2}\sum_{n=1}^{\infty} n\, c_n(x)\, e_{\Phi_\rho n}(t) \int_0^t e_{\Phi_\rho n}(-\tau) \int_0^L s_n(u) g_{\Phi_\rho}(u,T) \frac{\partial \Phi_{I_\rho i-1}(u,\tau)}{\partial u} d u\, d\tau,\ i\ge 1.$$

Equations for initial-order approximation of concentration of dopant $C_{00}(x,t)$ and corrections for them $C_{ij}(x,t)$ ($i \ge 1$, $j \ge 1$), boundary and initial conditions for the above equations could be written as

$$\frac{\partial C_{00}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{00}(x,t)}{\partial x^2};$$

$$\frac{\partial C_{i0}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{i0}(x,t)}{\partial x^2} + D_{0L} \frac{\partial}{\partial x}\left[g_L(x,T) \frac{\partial C_{i-10}(x,t)}{\partial x}\right],\ i \ge 1;$$

$$\frac{\partial C_{01}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{01}(x,t)}{\partial x^2} + D_{0L} \frac{\partial}{\partial x}\left[\frac{C_{00}^{\gamma}(x,t)}{P^{\gamma}(x,T)} \frac{\partial C_{00}(x,t)}{\partial x}\right];$$

$$\frac{\partial C_{02}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{02}(x,t)}{\partial x^2} + D_{0L} \frac{\partial}{\partial x}\left[C_{01}(x,t) \frac{C_{00}^{\gamma-1}(x,t)}{P^{\gamma}(x,T)} \frac{\partial C_{00}(x,t)}{\partial x}\right] +$$
$$+ D_{0L} \frac{\partial}{\partial x}\left[\frac{C_{00}^{\gamma}(x,t)}{P^{\gamma}(x,T)} \frac{\partial C_{01}(x,t)}{\partial x}\right];$$

$$\frac{\partial C_{02}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{02}(x,t)}{\partial x^2} + D_{0L} \frac{\partial}{\partial x}\left[C_{01}(x,t) \frac{C_{00}^{\gamma-1}(x,t)}{P^{\gamma}(x,T)} \frac{\partial C_{00}(x,t)}{\partial x}\right] +$$
$$+ D_{0L} \frac{\partial}{\partial x}\left[\frac{C_{00}^{\gamma}(x,t)}{P^{\gamma}(x,T)} \frac{\partial C_{01}(x,t)}{\partial x}\right];$$

$$\frac{\partial C_{11}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{11}(x,t)}{\partial x^2} + D_{0L} \frac{\partial}{\partial x}\left[C_{10}(x,t) \frac{C_{00}^{\gamma-1}(x,t)}{P^{\gamma}(x,T)} \frac{\partial C_{00}(x,t)}{\partial x}\right] +$$

$$+D_{0L}\frac{\partial}{\partial x}\left[\frac{C_{00}^{\gamma}(x,t)}{P^{\gamma}(x,T)}\frac{\partial C_{10}(x,t)}{\partial x}\right]+D_{0L}\frac{\partial}{\partial x}\left[g_L(x,T)\frac{\partial C_{01}(x,t)}{\partial x}\right];$$

for finite source of dopant

$$\left.\frac{\partial C_{ij}(x,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial C_{ij}(x,t)}{\partial x}\right|_{x=L}=0,\ i\geq 0,\ j\geq 0;\ C_{00}(x,0)=f_C(x),\ C_{ij}(x,0)=0,\ i\geq 1,\ j\geq 1.$$

for infinite source of dopant

$$C_{00}(0,t)=C_0,\ \left.\frac{\partial C_{ij}(x,t)}{\partial x}\right|_{x=L}=0,\ C_{ij}(x>0,t)=0,\ C_{00}(0,0)=C_0,\ C_{ij}(x>0,0)=0,\ i\geq 0,\ j\geq 0.$$

Solutions of the above equations with account boundary and initial conditions by standard Fourier approach takes the form

for finite source of dopant

$$C_{00}(x,t)=\frac{F_{0C}}{L}+\frac{2}{L}\sum_{n=1}^{\infty}F_{nC}c_n(x)e_{nC}(t),$$

where $e_{nC}(t)=\exp\left(-\pi^2n^2D_{0C}t/L^2\right)$, $F_{nC}=\int_0^L c_n(u)f_C(u)\,du$;

$$C_{i0}(x,y,z,t)=-\frac{2\pi}{L^2}\sum_{n=1}^{\infty}nF_{nC}c_n(x)e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^L s_n(u)\,g_L(u,T)\frac{\partial C_{i-10}(u,\tau)}{\partial u}\,du\,d\tau,\ i\geq 1;$$

$$C_{01}(x,t)=-\frac{2\pi}{L^2}\sum_{n=1}^{\infty}nF_{nC}c_n(x)\,e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^L s_n(u)\frac{C_{00}^{\gamma}(u,\tau)}{P^{\gamma}(u,T)}\frac{\partial C_{00}(u,\tau)}{\partial u}\,du\,d\tau;$$

$$C_{02}(x,t)=-\frac{2\pi}{L^2}\sum_{n=1}^{\infty}nF_{nC}\,c_n(x)\,e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^L s_n(u)\,C_{01}(u,\tau)\frac{C_{00}^{\gamma-1}(u,\tau)}{P^{\gamma}(u,T)}\frac{\partial\,C_{00}(u,\tau)}{\partial\,u}\,du\,d\tau-$$

$$-\frac{2\pi}{L^2}\sum_{n=1}^{\infty}nF_{nC}c_n(x)\,e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^L s_n(u)\,C_{01}(u,\tau)\frac{C_{00}^{\gamma-1}(u,\tau)}{P^{\gamma}(u,T)}\frac{\partial C_{00}(u,\tau)}{\partial u}\,du\,d\tau-$$

$$-\frac{2\pi}{L^2}\sum_{n=1}^{\infty}nF_{nC}c_n(x)\,e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^L s_n(u)\frac{C_{00}^{\gamma}(u,\tau)}{P^{\gamma}(u,T)}\frac{\partial C_{01}(u,\tau)}{\partial u}\,du\,d\tau;$$

$$C_{11}(x,t)=-\frac{2\pi}{L^2}\sum_{n=1}^{\infty}nF_{nC}c_n(x)e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^L s_n(u)g_L(u,T)\frac{\partial C_{01}(u,\tau)}{\partial\,u}\,du\,d\tau-$$

$$-\frac{2\pi}{L^2}\sum_{n=1}^{\infty}nF_{nC}c_n(x)\,e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^L s_n(u)\frac{C_{00}^{\gamma}(u,\tau)}{P^{\gamma}(u,T)}\frac{\partial C_{10}(u,\tau)}{\partial u}\,du\,d\tau-$$

$$-\frac{2\pi}{L^2}\sum_{n=1}^{\infty}nF_{nC}c_n(x)e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^L s_n(u)\,C_{10}(u,\tau)\frac{C_{00}^{\gamma-1}(u,\tau)}{P^{\gamma}(u,T)}\frac{\partial C_{00}(u,\tau)}{\partial\,u}\,du\,d\tau.$$

for infinite source of dopant

$$C_{00}(x,t)=C_0\left\{1+\frac{2}{\pi}\sum_{n=0}^{\infty}\frac{1}{n+0.5}\sin\left[\frac{\pi(n+0.5)\,x}{L}\right]e_{n+0.5}(t)\right\},$$

$$C_{10}(x,t)=\frac{2\pi D_0C_0}{L^3}\sum_{n=0}^{\infty}(n+0.5)\sin\left[\frac{\pi(n+0.5)\,x}{L}\right]e_{n+0.5}(t)\sum_{m=0}^{\infty}\int_0^t e_{n+0.5}(-u)\,e_{m+0.5}(-u)\times$$

$$\times\left[H_{n+m+1}(u)+H_{n-m}(u)\right]du,$$

$$C_{20}(x,t)=-\frac{2D_0^2\pi^3C_0}{L^6}\sum_{k=0}^{\infty}(k+0.5)\sin\left[\frac{\pi(k+0.5)\,x}{L}\right]e_{k+0.5}(t)\sum_{n=0}^{\infty}(n+0.5)^2\sum_{m=1}^{\infty}\int_0^t e_{k+0.5}(-u)\times$$

$$\times\,e_{n+0.5}(u)\left[H_{n-k}(u)+H_{n+k+1}(u)\right]\int_0^u e_{n+0.5}(-\tau)e_{m+0.5}(\tau)\left[H_{n-m}(\tau)+H_{n+m+1}(\tau)\right]d\tau\,du,$$

where $H_n(t)=\int\limits_0^L \eta(y,t)\sin\left[\frac{\pi(n+0.5)\,y}{L}\right]d\,y$,

$$C_{01}(x,t)=-\gamma\,\alpha_1-\alpha_2\,,$$

where $\alpha_1=\frac{2C_0}{\pi^3}\sum\limits_{n=0}^{\infty}(n+0.5)^2\sin\left[\frac{\pi(n+0.5)\,x}{L}\right]e_{n+0.5}(t)\sum\limits_{k=0}^{\infty}\frac{1}{k+0.5}\sum\limits_{m=0}^{\infty}\frac{e_{km}(t)-e_{n+0,5}(t)}{nn-mm-kk}\times$

$\times\left[\frac{1}{mm-(n-k)^2}-\frac{1}{(m+0.5)-(n+k+1)^2}\right]$, $e_{km}(t)=(t)\ \ e_{k+0.5}(t)\ \ e_{m+0.5}(t)$,

$$\alpha_2=\begin{cases}0,\gamma<3\\ \frac{C_0}{\pi^5}\sum\limits_{n=0}^{\infty}(n+0.5)^3\sin\left[\frac{\pi(n+0.5)\,x}{L}\right]e_{n+0.5}(t)\sum\limits_{k=0}^{\infty}\frac{1}{k+0.5}\sum\limits_{l=0}^{\infty}\frac{1}{l+0.5}\sum\limits_{i=0}^{\infty}\frac{1}{i+0.5}\sum\limits_{j=0}^{\infty}\frac{1}{j+0.5}\times\end{cases}$$

$$\times\left[\frac{1}{nn-(k-l+i-j)^2}+\frac{1}{(n+0.5)^2-(k-l-i-j)^2}-\frac{1}{(n+0.5)^2-(k-l+i+j+1)^2}-\right.$$

$$-\frac{1}{(n+0.5)^2-(k-l-i-j+1)^2}-\frac{1}{(n+0.5)^2-(k+l+i-j+1)^2}-$$

$$\left.-\frac{1}{(n+0.5)^2-(i-j-k-l+1)^2}+\frac{1}{(n+0.5)^2-(i+j+k+l-2)^2}+\frac{1}{(n+0.5)^2-(i+j-k-l)^2}\right]\times$$

$$\times\left[\frac{e_{klij}(t)-e_{n+0.5}(t)}{(n+0.5)^2-(k+0.5)^2-(l+0.5)^2-(i+0.5)^2-(j+0.5)^2}\right],\ \gamma\ =\ 3,$$

$$C_{02}(x,t)=-\gamma^2\alpha_3-\alpha_4\,,$$

where $\alpha_3=\frac{4C_0}{\pi^5}\sum\limits_{n=0}^{\infty}\sin\left[\frac{\pi(n+0.5)\,x}{L}\right]\sum\limits_{k=1}^{\infty}\sum\limits_{l=0}^{\infty}\frac{(l+0.5)^3}{(l+0.5)-(n-k)^2}\sum\limits_{m=1}^{\infty}\frac{(n-k)^2-(n+k+1)^2}{(l+0.5)-(n+k+1)^2}\times$

$$\times\frac{(n+0.5)^2}{k+0.5}\sum\limits_{i=0}^{\infty}\sum\limits_{j=0}^{\infty}\frac{(l+i+1)^2-(l-i)^2}{(l+0.5)^2-(i+0.5)^2-(j+0.5)^2}\frac{1}{\left[(j+0.5)^2-(l+i+1)^2\right]\left[(j+0.5)^2-(l-i)^2\right]}\times$$

$$\times\frac{e_{n+0.5}(t)}{i+0.5}\left[\frac{e_{klj}(t)-e_{n+0.5}(t)}{(n+0.5)^2-(k+0.5)^2-(i+0.5)^2-(j+0.5)^2}-\frac{e_{kl}(t)-e_{n+0.5}(t)}{(n+0.5)^2-(k+0.5)^2-(l+0.5)^2}\right],$$

$$\alpha_4=\begin{cases}0,\gamma=1,\gamma=2\\ \frac{3C_0}{\pi^7}\sum\limits_{n=0}^{\infty}(n+0.5)^4\sin\left[\frac{\pi(n+0.5)x}{L}\right]e_{n+0.5}(t)\sum\limits_{i=0}^{\infty}\sum\limits_{j=0}^{\infty}\frac{(j+0.5)^2}{i+0.5}\left[\frac{1}{(n+0.5)^2-(i-j)^2}-\right.\end{cases}$$

$$\left.-\frac{1}{(n+0.5)^2-(i+j+1)^2}\right]\sum\limits_{k=0}^{\infty}\frac{1}{k+0.5}\sum\limits_{l=0}^{\infty}\frac{1}{l+0.5}\sum\limits_{m_1=0}^{\infty}\frac{1}{m_1+0.5}\sum\limits_{m_2=0}^{\infty}\left[\frac{1}{(j+0.5)^2-(k-l+m_1-m_2)^2}-\right.$$

$$-\frac{1}{(j+0.5)^2-(k-l-m_1-m_2-1)^2}+\frac{1}{(j+0.5)^2-(k-l-m_1+m_2)^2}-$$

$$-\frac{1}{(j+0.5)^2-(k-l+m_1+m_2)^2}-\frac{1}{(j+0.5)^2-(k+l+m_1-m_2-1)^2}-$$

$$-\frac{1}{(j+0.5)^2-(k+l-m_1+m_2-1)^2}+\frac{1}{(j+0.5)^2-(k+l-m_1-m_2)^2}+$$

$$\left.+\frac{1}{(j+0.5)^2-(k+l+m_1+m_2+2)^2}\right]\frac{1}{m_2+0.5}\left[-\frac{e_{ij}(t)-e_n(t)}{(n+0.5)^2-(i+0.5)^2-(j+0.5)^2}+\right.$$

$$+\frac{e_{iklm_1m_2}(t)-e_n(t)}{(n+0.5)^2-(i+0.5)^2-(j+0.5)^2-(k+0.5)^2-(l+0.5)^2-(m_1+0.5)^2-(m_2+0.5)^2}\Bigg]+$$

$$+\frac{4C_0}{\pi^7}\sum_{n_1=0}^{\infty}(n_1+0.5)^3\sin\left[\frac{\pi(n_1+0.5)x}{L}\right]\sum_{n_2=0}^{\infty}\frac{1}{n_2+0.5}\sum_{n_3=0}^{\infty}\left[\frac{1}{(n_1+0.5)^2-(n_2+n_3+n_4-n_5+1)^2}-\right.$$

$$-\frac{1}{(n_1+0.5)^2-(n_2+n_3-n_4+n_5+1)^2}-\frac{1}{(n_1+0.5)^2-(n_2-n_3+n_4+n_5+1)^2}-$$

$$-\frac{1}{(n_1+0.5)^2-(n_2-n_3-n_4-n_5-1)^2}+\frac{1}{(n_1+0.5)^2-(n_2+n_3-n_4-n_5)^2}+$$

$$\left.+\frac{1}{(n_1+0.5)^2-(n_2+n_3+n_4+n_5+2)^2}\right]\frac{1}{n_3+0.5}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\frac{1}{(n_5-0.5)^2-(k-0.5)^2-(m-0.5)^2}\times$$

$$\times\frac{1}{k+0.5}\left[\frac{1}{(m-0.5)^2-(n_5-k)^2}-\frac{1}{(m-0.5)^2-(n_5+k+1)^2}\right]\left[-\frac{e_{n_5}(t)-e_{n_1}(t)}{(n_1-0.5)^2-(n_5-0.5)^2}+\right.$$

$$\left.-\frac{e_{n_2n_3n_4km}(t)-e_{n_1}(t)}{(n_1-0.5)^2-(n_2-0.5)^2-(n_3-0.5)^2-(n_4-0.5)^2-(k-0.5)^2-(m-0.5)^2}\right].$$

$$C_{11}(x,t)=\alpha_5+2\alpha_6+2\alpha_7-2\alpha_8+2\alpha_9+\alpha_{10}+4\alpha_{11},$$

where $\alpha_5=\frac{2\gamma D_0C_0}{\pi L^3}\sum_{l=0}^{\infty}(l+0.5)\sin\left[\frac{\pi(l+0.5)x}{L}\right]e_{l+0.5}(t)\sum_{n=0}^{\infty}(n+0.5)^3\sum_{k=0}^{\infty}\frac{1}{k+0.5}\times$

$$\times\sum_{m=0}^{\infty}\frac{1}{(n+0.5)^2-(k+0.5)^2-(n+0.5)^2}\left[\frac{1}{(m+0.5)^2-(n-k)^2}-\frac{1}{(m+0.5)^2-(n+k+1)^2}\right]\times$$

$$\times\int_0^t e_{l+0.5}(-u)\left[e_{k+0.5}(u)e_{m+0.5}(u)-e_{n+0.5}(-u)\right]\left[H_{l-n}(u)+H_{l+n+1}(u)\right]du,$$

$$\alpha_6=\frac{\gamma D_0^2C_0}{L^5}\sum_{n=0}^{\infty}(n+0.5)^2\sin\left[\frac{\pi(n+0.5)x}{L}\right]e_{n+0.5}(t)\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left[\frac{1}{(l+0.5)^2-(n-k)^2}-\frac{1}{(l+0.5)^2-(n+k+1)^2}\right]\times$$

$$\times(l+0.5)^2(k+0.5)^3\sum_{m=0}^{\infty}\int_0^t e_{n+0.5}(-u)\,e_{k+0.5}(u)e_{l+0.5}(u)\int_0^u e_{l+0.5}(-\tau)\,e_{m+0.5}(\tau)\left[H_{l-m}(\tau)+H_{l+m+1}(\tau)\right]d\tau\,du,$$

$$\alpha_7=\frac{\pi D_0C_0}{L^3}\sum_{n=0}^{\infty}(n+0.5)\,e_{n+0.5}(t)\sin\left[\frac{\pi(n+0.5)x}{L}\right]\sum_{m=0}^{\infty}\int_0^t e_{n+0.5}(-u)e_{m+0.5}(u)\left[H_{m-n}(u)+H_{m+n+1}(u)\right]du$$

,

$$\alpha_8=\frac{\gamma\pi D_0C_0}{L^3}\sum_{n=0}^{\infty}(n+0.5)\sin\left[\frac{\pi(n+0.5)x}{L}\right]e_{n+0.5}(t)\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{k+0.5}\int_0^t e_{n+0.5}(-u)e_{km}(u)\left[H_{n+k-m+0.5}(u)+\right.$$

$$\left.+H_{n-k+m+0.5}(u)-H_{n-k-m-0.5}(u)-H_{n+k+m+1.5}(u)\right]du,$$

$$\alpha_9=\begin{cases}0,\gamma=1\\ \frac{\gamma D_0C_0}{\pi L^3}\sum_{n=0}^{\infty}(n+0.5)e_{n+0.5}(t)\sin\left[\frac{\pi(n+0.5)x}{L}\right]\sum_{k=0}^{\infty}\frac{1}{k+0.5}\sum_{l=0}^{\infty}\frac{1}{l+0.5}\sum_{m=0}^{\infty}\int_0^t e_m(-u)e_{klm}(u)\left[I_{k-l-n+m}(u)+\right.\end{cases}$$

$+I_{k-l+n-m}(u)+I_{k-l-n-m-1}(u)+I_{k-l+n+m+1}(u)-I_{k-l-n-m-1}(u)-I_{k+l+n-m+1}(u)-I_{m-k-l+n-}(u)-I_{m+k+l+n+2}(u)]\,d\,u$, $\gamma>2$,

$$I_n(t)=\int_0^L\eta(y,t)\cos\left[\pi(n+0.5)yL^{-1}\right]dy,$$

$$\alpha_{10}=\begin{cases}0,\gamma<3\\ \frac{2D_0C_0}{\pi^3L^3}\sum_{i=0}^{\infty}(i+0.5)e_{i+0.5}(t)\sin\left[\frac{\pi(i+0.5)\,x}{L}\right]\sum_{n=0}^{\infty}(n+0.5)^2\int_0^t\left[I_{i-n}(u)+I_{i+n+1}(u)\right]\sum_{k=0}^{\infty}\frac{1}{k+0.5}\times\end{cases}$$

$$\times\sum_{m_1=0}^{\infty}\frac{1}{m_1+0.5}\sum_{m_2=0}^{\infty}\left[\frac{1}{(n+0.5)^2-(k-l+m_1-m_2)^2}+\frac{1}{(n+0.5)^2-(k-l-m_1+m_2)^2}-\right.$$

$$-\frac{1}{(n+0.5)^2-(k-l+m_1+m_2+1)^2}-\frac{1}{(n+0.5)^2-(k-l-m_1-m_2-1)^2}-$$

$$-\frac{1}{(n+0.5)^2-(k+l+m_1+m_2+1)^2}-\frac{1}{(n+0.5)^2-(k+l-m_1+m_2+1)^2}+$$

$$\left.+\frac{1}{(n+0.5)^2-(k+l-m_1-m_2)^2}+\frac{1}{(n+0.5)^2-(k+l+m_1+m_2+2)^2}\right]\frac{1}{m_2+0.5}\sum_{l=0}^{\infty}\frac{1}{l+0.5}\times$$

$$\times\frac{e_i(-u)e_{klm_1m_2}(u)-e_i(-u)e_n(u)}{(n+0.5)^2-(k+0.5)^2-(l+0.5)^2-(m_1+0.5)^2-(m_2+0.5)^2}\,du\,,\ \gamma=3,$$

$$\alpha_{01}=\begin{cases}0,\gamma<3\\ \frac{D_0^2C_0}{\pi^3L^5}\sum_{i=0}^{\infty}(i+0.5)^3e_{i+0.5}(t)\sin\left[\frac{\pi(i+0.5)\,x}{L}\right]\sum_{j=0}^{\infty}\frac{1}{j+0.5}\sum_{k=0}^{\infty}\frac{1}{k+0.5}\sum_{l=0}^{\infty}\frac{1}{l+0.5}\sum_{n=0}^{\infty}(n+0.5)\times\end{cases}$$

$$\times\left[\frac{1}{(i+0.5)^2-(j-k+l-n)^2}+\frac{1}{(i+0.5)^2-(j-k-l+n)^2}-\frac{1}{(i+0.5)^2-(j+k-l+n+1)^2}-\right.$$

$$\left.-\frac{1}{(i+0.5)^2-(j+k-l-n+1)^2}+\frac{1}{ii-(j+k-l-n)^2}-\frac{1}{ii-(j+k+l+n+1)^2}\right]\times$$

$$\times\sum_{m=0}^{\infty}\int_0^t e_{i+0.5}(-u)e_{jk\ln}(u)\int_0^u e_{n+0,5}(-\tau)e_m(\tau)\left[I_{n-m}(\tau)+I_{n+m+1}(\tau)\right]d\tau\,du\,,\ \gamma=3,$$

$$\alpha_{11}=\begin{cases}0,\gamma<3\\ \frac{D_0^2C_0}{\pi^2L^3}\sum_{n=0}^{\infty}(n+0.5)e_{n+0.5}(t)\sin\left[\frac{\pi\,(n+0.5)x}{L}\right]\sum_{k=0}^{\infty}\frac{1}{k+0.5}\sum_{l=0}^{\infty}\frac{1}{l+0.5}\sum_{m=0}^{\infty}\frac{1}{m+0.5}\sum_{i=0}^{\infty}\int_0^t e_{n+0.5}(-u)\times\end{cases}$$

$$\times e_{klmi}(u)\left[H_{k-n+i-l+m+0.5}(u)+H_{k-n-i+l-m+0.5}(u)+H_{k-n+i+l-m+0.5}(u)+H_{k+n-i-l+m+0.5}(u)-H_{k+n-i-l+m-0.5}(u)-\right.$$

$$-H_{k-n+i+l-m+1.5}(u)-H_{k-n+i-l+m-0.5}(u)-H_{k+n-i+l-m+1.5}(u)+H_{k+n+i-l+m+1.5}(u)+H_{k-n-i+l-m-0.5}(u)+$$

$$+H_{k-n-i-l+m-0.5}(u)+H_{k+n+i+l-m+1.5}(u)-H_{k+n+i-l-m+0.5}(u)-H_{k+n-i+l+m+0.5}(u)-H_{k-n-i-l-m-1.5}(u)-$$

$$\left.-H_{k+n+i+l+m+2.5}(u)\right]d\ u\,.$$