BIFURCATION ANALYSIS AND MULTIOBJECTIVE NONLINEAR MODEL PREDICTIVE CONTROL OF A CLIMATE DYNAMIC MODEL

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ABSTRACT

Rigorous bifurcation analysis and multi-objective nonlinear Model Predictive Control Calculations are performed on a climate model involving atmosphere and ocean dynamics. The Bifurcation Analysis showed the existence of unwanted oscillation causing Hopf Bifurcations while the Multiobjective Nonlinear Model Predictive Control calculations resulted in the control profiles exhibiting spikes. Both the Hopf bifurcations and the spikes were eliminated using an activation factor involving the tanh function. Bifurcation analysis was performed using the MATLAB software MATCONT while the multi-objective nonlinear model predictive control was performed with the optimization language PYOMO. Numerical results are presented and explained.

KEYWORDS

Climate, Atmosphere, Optimization, Control

1. INTRODUCTION

Climate dynamics is extremely complex and several models have been developed to understand and control the highly nonlinear behavior of the air motion in and temperature gradients in the atmosphere and the ocean. In this work, multiobjective nonlinear model predictive control and bifurcation analysis are performed on a climate model that involves convective motion and temperature gradient in a climate model involving atmosphere and ocean dynamics. This paper is organized as follows. The background material is first presented followed by the model description and details of the bifurcation analysis and the multiobjective nonlinear model predictive control calculation procedures. The results and discussion are then presented.

2. BACKGROUND

The increase in global warming and the devastating effect of Hurricanes has motivated research involving strategies to control and even attempt to modify the climate. Several books (Shepherd et al 2009; Trenberth 1992; Washington, 2005 Olver and Bridgman 2014; Barry and Hall-McKim, 2014; Weart, 2014; Dennis, 1980; Fleming 2010, Dijkstra, 2013, Summerhayes, 2015) have been published discussing the uncertain nonlinear patterns and the need to control the global climate. Hoffmann (2002) discussed strategies to control the global weather. Curic et al (2007) used the cloud-resolving mesoscale model to study cloud seeding impact on precipitation. Mitchell and Finnegan (2009) investigated the possibility of modifying cirrus clouds to reduce global warming. Significant research on climate change was performed by Garstang et al (2005), Bengtsson(2006), Crutzen (2006), Wigley (2006), MacCracken(2009) Robock et al (2009), McClellan et al (2012)and Guo et al (2015). Soldatenko and co-workers (2014, 2015, 2017) have studied nonlinear dynamics and performed optimal control studies for mathematical models of climate manipulation.

3. MOTIVATION AND OBJECTIVES

Almost all optimization and optimal control of mathematical models of climate dynamics involve single-objective optimization. This work aims to perform multiobjective nonlinear model predictive control in conjunction with bifurcation analysis of a mathematical model involving climate dynamics. The model used is described in Soldatenko(2017) where the Earth's Climate System (ECS) considers both and atmosphere.

4. MODEL DESCRIPTION

The coupled nonlinear model, Soldatenko (2017) consists of the atmosphere and ocean components. x_A, y_A, z_A represent the intensity of convective motion and horizontal and vertical temperature gradients in the atmosphere and x_B, y_B, z_B represent the same variables in the ocean. The dynamic model equations are

$$
\begin{aligned}
\frac{dx_A}{dt} &= \sigma(y_A - x_A) - c(ax_B + k) \\
\frac{dy_A}{dt} &= rx_A - y_A - x_A z_A + c(ay_B + k) \\
\frac{dz_A}{dt} &= x_A y_A - bz_A + cz_B \\
\frac{dx_B}{dt} &= \lambda\sigma(y_B - x_B) - c(ax_A + k) \\
\frac{dy_B}{dt} &= \lambda(rx_B - y_B - ax_B z_B) + c(ay_A + k) \\
\frac{dz_B}{dt} &= \lambda(ax_B y_B - bz_B) - cz_A
\end{aligned}
\tag{1}
$$

The parameter values are $a = 1; k = 0; b = 8/3; \sigma = 10; r = 28; \lambda = 0.1$

5. BIFURCATION ANALYSIS

There has been a lot of work in chemical engineering involving bifurcation analysis throughout the years. The existence of multiple steady-states and oscillatory behavior in chemical processes has led to a lot of computational and analytical work to explain the causes for these nonlinear phenomena. Multiple steady states are caused by the existence of branch and limit points while oscillatory behavior is caused by the existence of Hopf bifurcations points.

One of the most commonly used software to locate limit points, branch points, and Hopf bifurcation points is MATCONT(Dhooge et al (2003¸2004) Govearts(2000), and Kuznetsov, [1998)] This software detects Limit points(LP), branch points(BP) and Hopf bifurcation points(HB). Consider an ODE system

$$\dot{x} = f(x, \beta) \qquad (2)$$

$x \in R^n$ Let the tangent plane at any point x be $[v_1, v_2, v_3, v_4, ...v_{n+1}]$. Define matrix A given by

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \cdots\cdots\cdots & \frac{\partial f_1}{\partial x_n} & \frac{\partial f_1}{\partial \beta} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \cdots\cdots\cdots & \frac{\partial f_2}{\partial x_n} & \frac{\partial f_2}{\partial \beta} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \frac{\partial f_n}{\partial x_3} & \frac{\partial f_n}{\partial x_4} \cdots\cdots\cdots & \frac{\partial f_n}{\partial x_n} & \frac{\partial f_n}{\partial \beta} \end{bmatrix} \qquad (3)$$

β is the bifurcation parameter. The matrix A can be written in a compact form as

$$A = [B| \quad \frac{\partial f}{\partial \beta}] \qquad (4)$$

The tangent surface must satisfy

$$Av = 0 \qquad (5)$$

For both limit and branch points the matrix B must be singular. For a limit point (LP) the n+1 th component of the tangent vector v_{n+1} = 0 and for a branch point (BP) the matrix $\begin{bmatrix} A \\ v^T \end{bmatrix}$ must be singular. For a Hopf bifurcation, the function $\det(2 f_x(x, \beta) @ I_n)$ should be zero. @ indicates the bialternate product while I_n is the n-square identity matrix. More details can be found in Kuznetsov (1998) and Govaerts (2000) . Sridhar [2011] used Matcont to perform bifurcation analysis on chemical engineering problems.

6. Multiobjective Nonlinear Model Predictive Control Algorithm

The multiobjective nonlinear model predictive control (MNLMPC) method was first proposed by Flores Tlacuahuaz(2012) and used by Sridhar(2019). This method does not involve weighting functions, nor does it impose additional constraints on the problem unlike the weighted function or the epsilon correction method(Miettinen, 1999). In a problem involving a set of ODE

$$\frac{dx}{dt} = F(x, u)$$

$$h(x,u) \le 0 \quad x^L \le x \le x^U; \quad u^L \le u \le u^U \qquad (6)$$

the MNLMPC method first solves dynamic optimization problems independently minimizing/maximizing each variable p_i individually. The minimization/maximization of p_i will lead to the values p_i^*. Then the optimization problem that will be solved is

$$
\begin{aligned}
&\min \sum_i \{p_i - p_i^*\}^2 \\
&\text{subject to } \frac{dx}{dt} = F(x,u); \quad h(x,u) \le 0 \\
&x^L \le x \le x^U; \quad u^L \le u \le u^U
\end{aligned}
\tag{7}
$$

This will provide the control values for various times. The first obtained control value is implemented and the remaining is discarded. This procedure is repeated until the implemented and the first obtained control values are the same. The optimization package in Python, Pyomo (Hart et al, 2017), where the differential equations are automatically converted to a Nonlinear Program (NLP) using the orthogonal collocation method will be used. The resulting nonlinear optimization problem was solved using the solvers IPOPT (Wächter And Biegler, 2006) and confirmed as a global solution with Baron (Tawarmalani, M. and N. V. Sahinidis 2005). To summarize the steps of the algorithm are as follows

1. Minimize/maximize p_i subject to the differential and algebraic equations that govern the process using Pyomo with IPOPT and Baron. This will lead to the value p_i^* at various time intervals t_i. The subscript *i* is the index for each time step.
2. Minimize $\sum_i \{p_i - p_i^*\}^2$ subject to the differential and algebraic equations that govern the process using Pyomo with IPOPT and Baron. This will provide the control values for various times.
3. Implement the first obtained control values and discard the remaining.
   Repeat steps 1 to 3 until there is an insignificant difference between the implemented and the first obtained value of the control variables or if the Utopia point is achieved. The Utopia point is when $p_i = p_i^*$ for all

7. RESULTS AND DISCUSSION

7.1 Bifurcation Oscillations and Control Profile Spikes

Bifurcation analysis of some models reveals the existence of oscillation causing Hopf bifurcations. Optimal control of some nonlinear ODE models produces spikes in the control profile. Both oscillations and control profile spikes are inconvenient as they impede optimization and make the implementation of control necessary control variables difficult. The tanh activation factor is used in neural networks (Szandała, 2020; Kamalov et al (2021) ; Dubey et al, 2022; . and in optimal control problems to eliminate spikes in the optimal control profile (Sridhar; 2023a, 2023b, 2023c).

Oscillations are similar to spikes and in the case of this climate model problem, oscillation causing Hopf bifurcations and spikes in the control profiles are obtained when bifurcation analysis and MNLMPC calculations are performed on the climate model equations (Eq. 1) . An activation factor involving the tanh function is used to eliminate both control profile spikes and

Hopf bifurcation oscillations. When the variable c (Eq. 1), which is both the bifurcation parameter and control variable was modified to (c tanh(c)/500) the oscillation causing Hopf bifurcation and the spikes in the control profile disappeared.

7.2 Bifurcation Analysis of Climate Model

The bifurcation analysis reveals 2 Hopf bifurcation points the co-ordinates of which are $[x_A, y_A, z_A, x_B, y_B, z_B, c]$ = (8.085944, 8.297009, 27.274397, 10.623396, 12.229902, 28.400484 0.198679) and (4.362063 9.939176 42.764834 27.027898 36.028865 34.255114 2.063465). When the bifurcation parameter c (Eq. 1) is modified to (c tanh(c)/500) both Hopf bifurcations disappear Fig. 1 shows both the bifurcation diagrams without and with the activation factor. The two Hopf bifurcation points when the activation factor was not used are indicated by the letter H in one of the curves.

7.3 MNLMC for Climate Model

First, the variables $\sum_0^{t_f} x_A, \sum_0^{t_f} y_A, \sum_0^{t_f} z_A, \sum_0^{t_f} x_B, \sum_0^{t_f} y_B, \sum_0^{t_f} z_B$ are individually minimized in both the cases, with and without the use of the tanh activation factor. In both the cases, each of the mimized values was 0. The multiobjective nonlinear model predictive control involved the minimization of

$$(\sum_0^{t_f} x_A - 0)^2 + (\sum_0^{t_f} y_A - 0)^2 + (\sum_0^{t_f} z_A - 0)^2 + (\sum_0^{t_f} x_B - 0)^2 + (\sum_0^{t_f} y_B - 0)^2, (\sum_0^{t_f} z_B - 0)^2$$

. In both the cases the multiobjective optimal control problem resulted in an optimal value of 0 (Utopia point). When no activation factor was used, the MNLMPC control value of c was 0.03896989876805711. When c was modified to (c tanh(c)/500) the MNLMPC control value of c obtained was 0.7495817434219589.

Figures 2a, 2b, and 2c show the variables and control profiles when the activation factor was not used. Figures 3a, 3b, and 3c show the same profiles when the activation factor was used. Fig. 2c shows distinct spikes in the control profile. The spikes disappeared when the activation factor was implemented (Fig. 3c).

The numerical results indicate that unwanted oscillation causing Hopf bifurcations and spikes in the control profiles were effectively eliminated when the tanh function activation factor was implemented.

8. CONCLUSIONS

The results of this work demonstrate the existence of oscillation causing Hopf bifurcation points in climate models considering atmospheric and ocean dynamics. When the multiobjective nonlinear model predictive control(MNLMPC) of this model was performed, spikes were observed in the control profile. Both the Hopf bifurcations and spikes were eliminated when the activation factor involving the tanh function was implemented. Bifurcation analysis and MNLMPC calculations for climate models with time delay would be future work.

Data Availability Statement

All data used is presented in the paper

Conflict of Interest

The author, Dr. Lakshmi N Sridhar has no conflict of interest.

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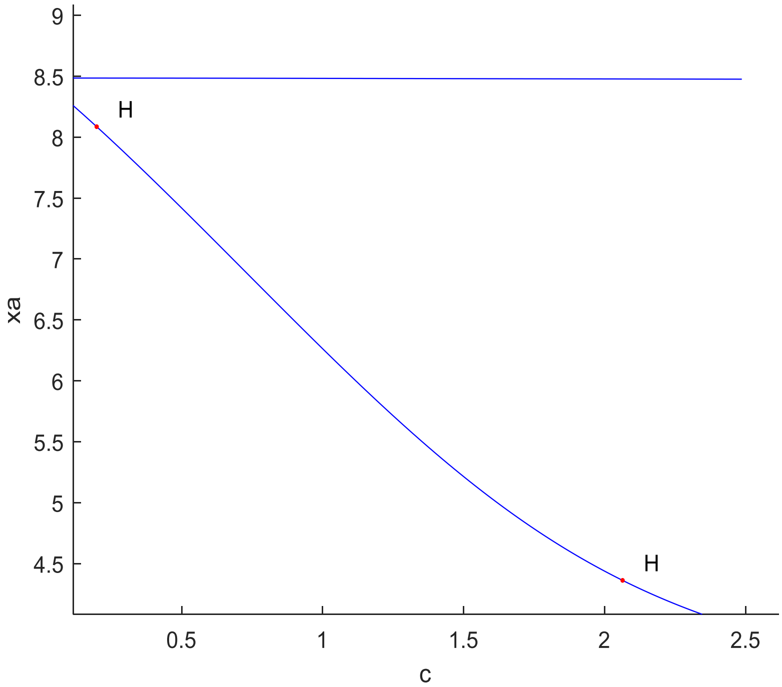

Fig. 1 (bifurcation curves without (two Hopf bifurcation points c =0.198679 ; c= 2.063465) and with activation factor(no Hopf Bifurcations)

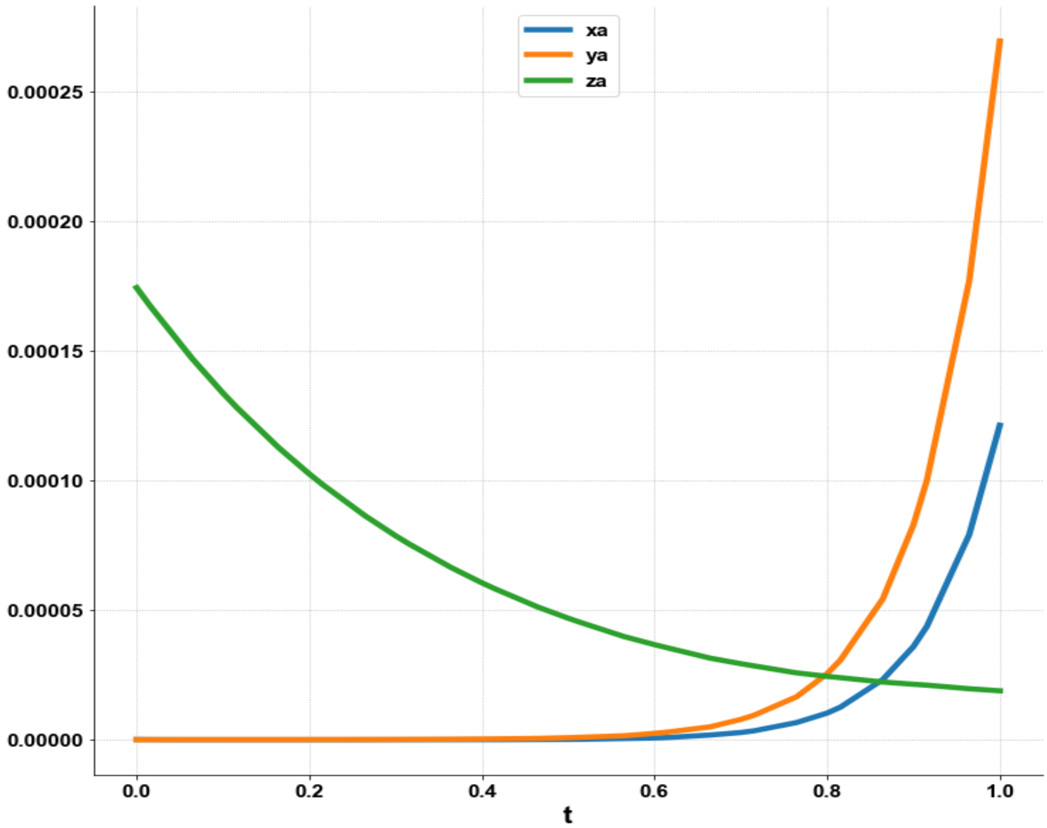

Fig. 2a (xa, ya,za without activation factor)

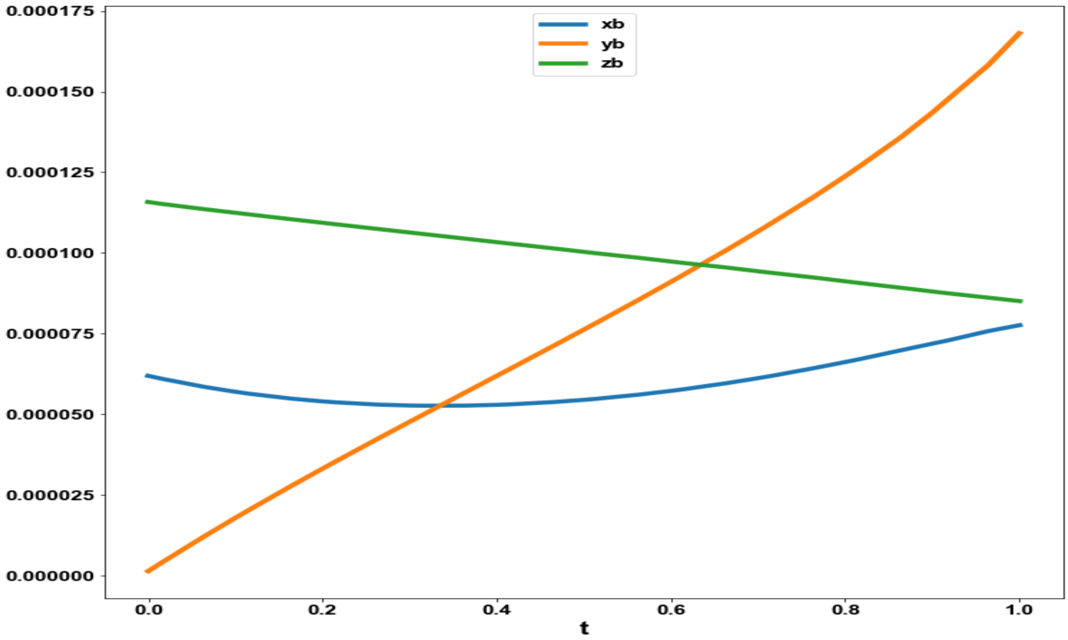

Fig. 2b (xb, yb,zb without activation factor)

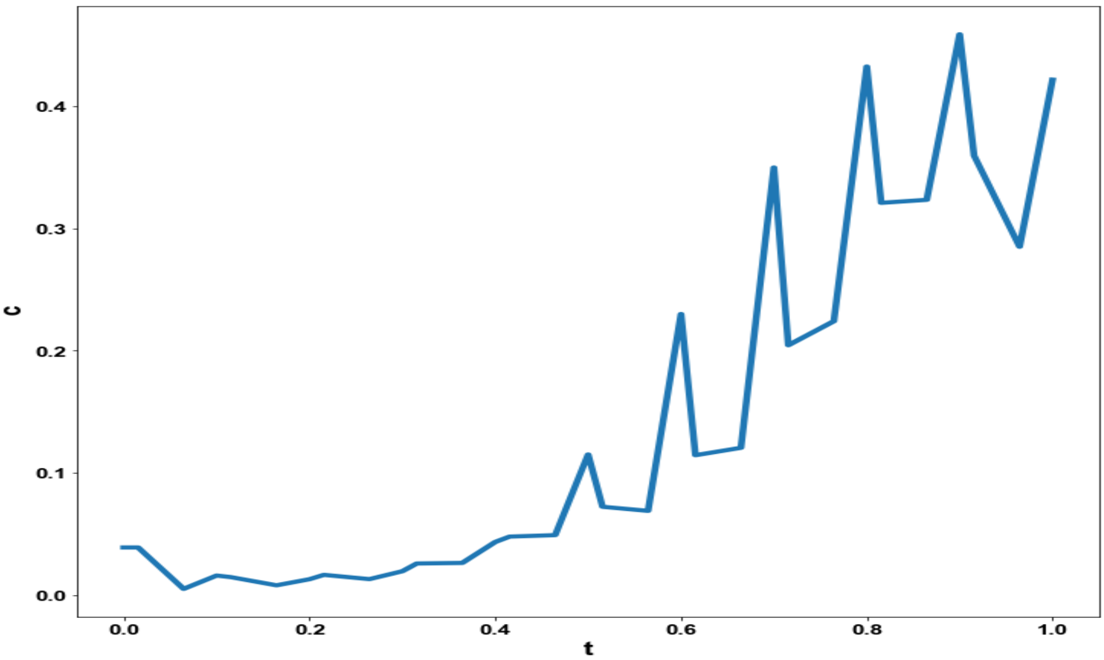

Fig. 2c (c vs t without activation factor; note the spikes;
MNLMPC value of c= 0.03896989876805711)

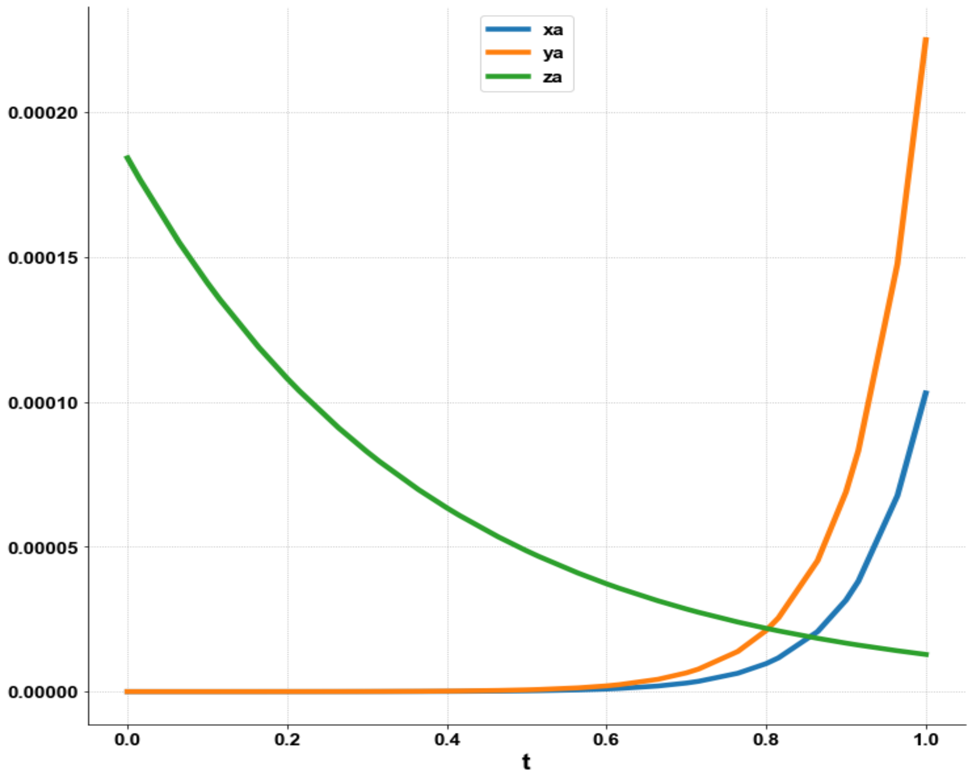

Fig. 3a (xa, ya,za with activation factor)

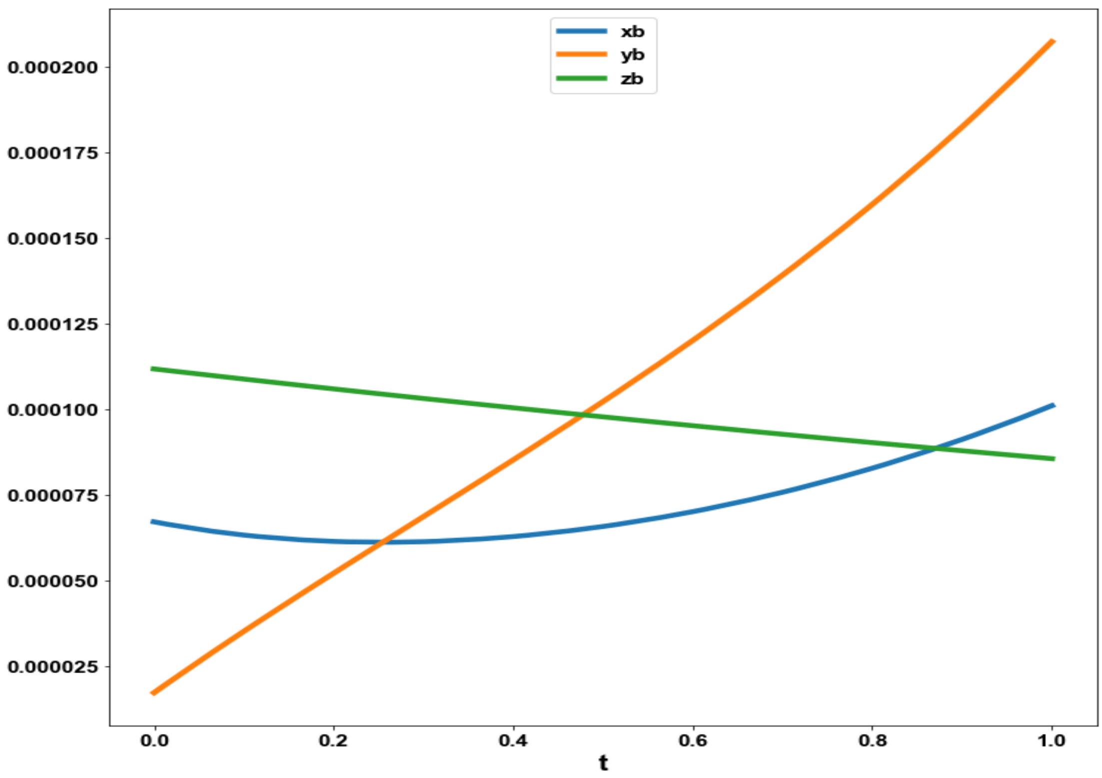

Fig. 3b (xb, yb,zb with activation factor)

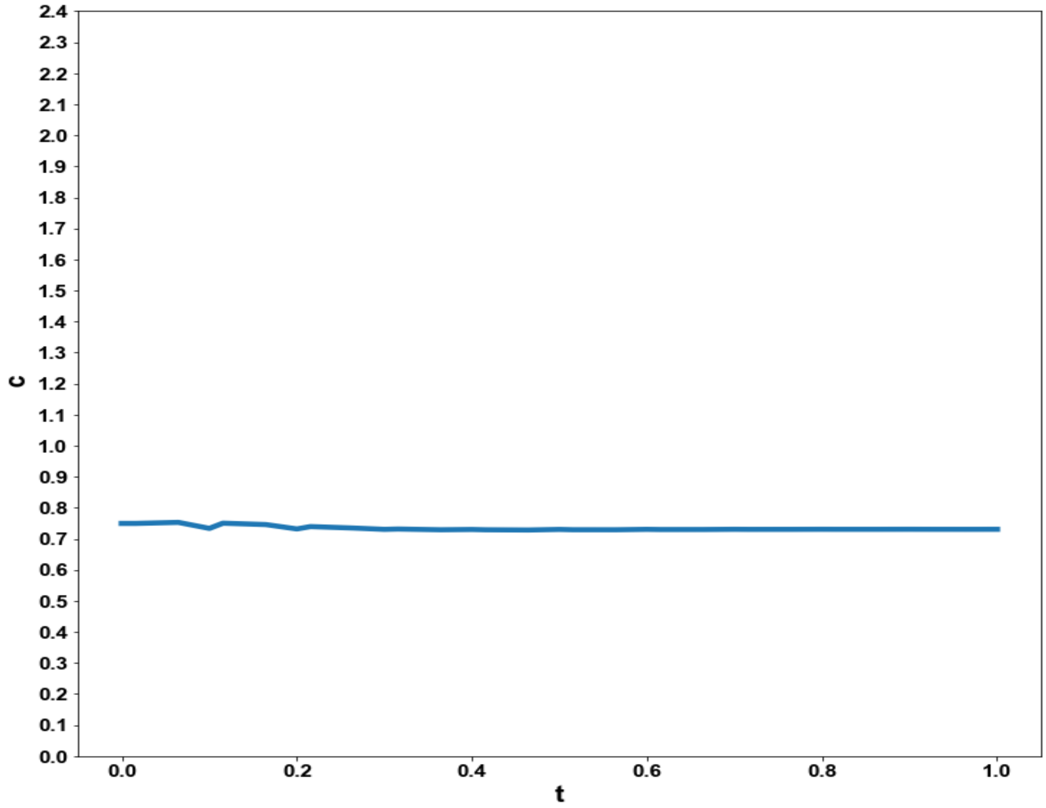

Fig. 3c (c .vs. t with activation factor (no spikes))
MNLMPC value of c= 0.7495817434219589