On Nano Semi Generalized B - Neighbourhood in Nano Topological Spaces

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Abstract

Objectives: *The primary objective of this study is to define and look into the neighbourhood, adherent points, and derived set in nano topological spaces for Nano semi generalized b (Nsgb) (closed/open) set, Further a real life application is examined using nano topology.*

Methods: *The definition of Nsgb-closed (open) sets is used to define the Nsgb-neighbourhood, Nsgb-adherent points, Nsgb-derived set. In addition, the indiscernibility equivalence class, which provide the core upon the basis of nano topology.*

Findings: *The properties of Nsgb-neighbourhood, Nsgb-adherent points and Nsgb-derived set for nano semi generalized b-set in a nano topological space are analyzed and discussed theorem on it. Using the attribute reduction in a complete information system, the key factor is identified for the causes of hair fall.*

Novelty: *The characteristics of Nsgb-neighbourhood, Nsgb-adherent points, Nsgb-derived set are given. The application of nano topology is provided to the real life situation.*

Keywords

Nano topology, Nsgb-neighbourhood, Nsgb-limit point, Nsgb-isolated point, Nsgb-derived set, core, basis.

1. Introduction

M. Lellis Thivagar [1] developed a new type of topology called nano topology in 2013 and described new types of closed sets like nano closed, nano semi-closed, nano pre-closed and so on using approximation spaces. Parimala, Indirani [2] coined nano b-open set. Kavipriya and Indira [3] developed Nsgb-closed set. Pawlak [4] developed rough set theory to structure and assess diverse sorts of data in data mining. Thivagar [5] used to find the deciding factors of chikungunya and diabetes using the concept of basis.

This paper is to define and look into the properties of Nsgb-neighbourhood, Nsgb-adherent points and Nsgb-derived set for nano semi generalized b-set in a nano topological space. In addition, using attribute reduction in a complete information system with the basis of nano topology to identify the root causes of hair fall.

Throughout the paper ($\mathcal{U}$, $\tau_{\mathcal{R}}$(S)) indicates the nano topological space where S $\subseteq \mathcal{U}$.

DOI : 10.5121/mathsj.2025.12103

2. METHODOLOGY

This present work is for the newly developed nano closed set namely nano semigeneralized b-closed set to discuss the ideas of neighbourhood, adherent points and derived set known as Nsgb-neighbourhood, Nsgb-adherent points and Nsgb-derived set in nano topological spaces with suitable examples using the basic definitions in nano topology. And also discuss the properties and theorems on it. Further to give application of nano topology with the help of its basis and attribute reduction in the collected data system.

Preliminaries:

Definition 2.1: [1] Assume $\mathcal{U}$ indicate a finite set referred as the Universe, $\mathcal{R}$ signify the equivalence relation on $\mathcal{U}$. Then $\mathcal{U}$ is partitioned into distinct equivalence classes and the elements are perceived to be indistinguishable. The **Approximation space** is given by ($\mathcal{U}$, $\mathcal{R}$). Let S ⊆ $\mathcal{U}$.

- The **Lower Approximation** of S ($L_{\mathcal{R}}(S)$) refers to the set of all elements in respect of $\mathcal{R}$ that could be certain categorized as S in respect of $\mathcal{R}$, $\mathcal{R}$(S) is the equivalence class generated by s in S.

$$L_{\mathcal{R}}(S) = \cup_{s\in U} \{R(S): R(S) \subseteq S\}$$

- The **Upper Approximation** of S ($U_{\mathcal{R}}(S)$) refers to the set of all elements in respect of $\mathcal{R}$ that could be possibly categorized as S in relation to $\mathcal{R}$.

$$U_{\mathcal{R}}(S) = \cup_{s\in U} \{R(S): R(S) \cap S \neq \emptyset\}$$

- The **Boundary Region** of S ($B_{\mathcal{R}}(S)$) refers the set of all elements in respect of $\mathcal{R}$ that could be labeled neither as S nor as not-S in respect of $\mathcal{R}$.

$$B_{\mathcal{R}}(S) = U_{\mathcal{R}}(S) - L_{\mathcal{R}}(S).$$

Definition 2.2: [1] Take an equivalence relation of $\mathcal{U}$ be $\mathcal{R}$, S ⊆ $\mathcal{U}$ and $\tau_{\mathcal{R}}(S) = \{\mathcal{U}, \emptyset, L_{\mathcal{R}}(S), U_{\mathcal{R}}(S), B_{\mathcal{R}}(S)\}$ must satisfy the given conditions.

i. $\mathcal{U}$ and $\emptyset$ belongs to $\tau_{\mathcal{R}}(S)$
ii. The arbitrary union of the components of $\tau_{\mathcal{R}}(S)$ belongs to $\tau_{\mathcal{R}}(S)$.
iii. The finite intersection of the components of $\tau_{\mathcal{R}}(S)$ belongs to $\tau_{\mathcal{R}}(S)$.

Then $\tau_{\mathcal{R}}(S)$ represents the **Nano topology** on $\mathcal{U}$ in respect of S and ($\mathcal{U}$, $\tau_{\mathcal{R}}(S)$) represents the Nano topological space. The components in $\tau_{\mathcal{R}}(S)$ are the nano open sets while its complements are the nano closed.

Definition 2.3: [1] The **basis** for $\tau_{\mathcal{R}}(S)$ is the set $\boldsymbol{\mathcal{B}} = \{\mathcal{U}, \emptyset, L_{\mathcal{R}}(S), B_{\mathcal{R}}(S)\}$.

Definition 2.4: [1, 2] A subset E of $\mathcal{U}$ is

(i) Nano semi-open (Ns-open): if E ⊆ Ncl(Nint(E))
(ii) Nano b-open (Nb-open): if E ⊆ Ncl(Nint(E)) ∪ Nint(Ncl(E)).

Definition 2.5: [6] Let u ϵ $\mathcal{U}$ in ($\mathcal{U}$, $\tau_{\mathcal{R}}$(S)). A subset N of $\mathcal{U}$ is termed as the **Nano-neighbourhood (N-nbhd) of u** if there exists a Nano-open set H where u ϵ H ⊆ N.

Definition 2.6: [3] A subset E of ($\mathcal{U}$, $\tau_{\mathcal{R}}$(S)) is **Nsgb-closed** if Nb-cl(E) ⊆ K, whenever E ⊆ K, where K is nano semi-open in $\mathcal{U}$. A subset E of ($\mathcal{U}$, $\tau_{\mathcal{R}}$(S)) is **Nsgb-open** if the complement $\mathcal{U}$ – E is Nsgb-closed.

Example 2.7: Take $\mathcal{U}$ = {n, o, p, q}, S = {o, q} ⊆ $\mathcal{U}$, $\mathcal{U}/\mathcal{R}$ = {{n}, {o}, {p, q}}, $\tau_{\mathcal{R}}$(S) = {$\mathcal{U}$, ∅, {o}, {o, p, q}, {p, q}}, Nsgb-closed = {$\mathcal{U}$, ∅, {n}, {o}, {p}, {q}, {n, o}, {n, p}, {n, q}, {p, q}, {n, o, p}, {n, o, q}, {n, p, q}}, Nsgb-open = {∅, $\mathcal{U}$, {o, p, q}, {n, p, q}, {n, o, q}, {n, o, p}, {p, q}, {o, q}, {o, p}, {n, o}, {q}, {p}, {o}}.

- The union of any Nsgb-open sets is Nsgb-open
- The intersection of any Nsgb-closed sets is Nsgb-closed.

3. RESULTS AND DISCUSSIONS:

Nsgb- Neighbourhood

Definition 3.1: Let u ϵ $\mathcal{U}$ in ($\mathcal{U}$, $\tau_{\mathcal{R}}$(S)). A subset N of $\mathcal{U}$ is termed as the **Nsgb-neighbourhood (Nsgb-nbhd) of u** if there exists a Nsgb-open set H where u ϵ H ⊆ N.

Thus every superset of Nsgb-open set that contains u is a Nsgb-neighbourhood of a point u.

- Each Nsgb-open set that contains u is a Nsgb-nbhd of u.
- A Nsgb-open set is a Nsgb-nbhd of each of its points. But Nsgb-nbhd of a point need not be Nsgb-open.
- If H is a Nsgb-open set that contains u, then it is a Nsgb-open neighbourhood of u.

Definition 3.2: The set of all Nsgb-neighbourhoods of a point u ϵ $\mathcal{U}$ is called **Nsgb-nbhd sysem of u** (Nsgb-$N_{(u)}$).

Example 3.3: Let $\mathcal{U}$ = {i, j, k, l}, $\mathcal{U}/\mathcal{R}$ = {{i}, {j}, {k}, {l}}, S = {i, j}, $\tau_{\mathcal{R}}$(S) = {$\mathcal{U}$, ∅, {i, j}}, Nsgb-open = {$\mathcal{U}$, ∅, {i}, {j}, {i, j}, {i, k}, {i, l}, {j, k}, {j, l}, {i, j, k}, {i, j, l}, {i, k, l}, {j, k, l}}. Take i ϵ $\mathcal{U}$, i ϵ {i, j} - a Nsgb-open set of $\mathcal{U}$, and {i, j} ⊂ {i, j, l}. ⇒ i ϵ {i, j} ⊂ {i, j, l} then {i, j, l} is Nsgb-nbhd of i.

<u>Nsgb-neighbourhood system of points of $\mathcal{U}$:</u>

For i ϵ $\mathcal{U}$, Nsgb-$N_{(i)}$ = {{i}, {i, j}, {i, k}, {i, l}, {i, j, k}, {i, j, l}, {i, k, l}, $\mathcal{U}$}
For j ϵ $\mathcal{U}$, Nsgb-$N_{(j)}$ = {{j}, {i, j}, {j, k}, {j, l}, {i, j, k}, {i, j, l}, {j, k, l}, $\mathcal{U}$}
For k ϵ $\mathcal{U}$, Nsgb-$N_{(k)}$ = {{i, k}, {j, k}, {i, j, k}, {i, k, l}, {j, k, l}, $\mathcal{U}$}
For l ϵ $\mathcal{U}$, Nsgb-$N_{(l)}$ = {{i, l}, {j, l}, {i, j, l}, {j, k, l}, {i, k, l}, $\mathcal{U}$}

Definition 3.4: A set N is **Nsgb-nbhd of a subset C** of $\mathcal{U}$ if C ⊆ H ⊆ N, where H is Nsgb-open of $\mathcal{U}$.

Example 3.5: Consider example 3.3, take C = {k, l} then {k, l} ⊆ {i, k, l} ⊆ {i, k, l}, $\mathcal{U}$. Here {i, k, l} and $\mathcal{U}$ are Nsgb-nbhds of {k, l}.

Nsgb-neighbourhood system of C:

For C = {i, j}, Nsgb-$N_{(\{i, j\})}$ = {{i, j}, {i, j, k}, {i, j, l}, $\mathcal{U}$}
For C = {k, l}, Nsgb-$N_{(\{k, l\})}$ = {{i, k, l}, {j, k, l}, $\mathcal{U}$}

Remark: Nsgb-nbhd is not necessary be Nsgb-open.

Example 3.6: Assume $\mathcal{U}$ = {i, j, k, l}, S = {k, l}, $\mathcal{U}/\mathcal{R}$ = {{i}, {k}, {j, l}}, $\tau_{\mathcal{R}}$(S) = {$\mathcal{U}$, ∅, {k}, {j, k, l}, {j, l}}, Nsgb-open = {$\mathcal{U}$, ∅, {j}, {k}, {l}, {i, k}, {j, l}, {j, k}, {k, l}, {i, j, k}, {j, k, l}, {i, k, l}}

i) {i, j, l} ⊂ $\mathcal{U}$ is a Nsgb-nbhd of a point l but not Nsgb-open since l ϵ {l} ⊂ {i, j, l} where {l} is Nsgb-open
ii) {i, j, l} ⊂ $\mathcal{U}$ is a Nsgb-nbhd of a subset {j, l} but not Nsgb-open since {j, l} ⊆ {j, l} ⊂ {i, j, l} where {j, l} is Nsgb-open

Theorem 3.7: A subset in ($\mathcal{U}$, $\tau_{\mathcal{R}}$(S)) is Nsgb-open iff it is a Nsgb-nbhd of each of its points.

Proof: ***Necessary Part:*** Take K ⊆ $\mathcal{U}$ be Nsgb-open. Then for each u ϵ K, ∃ a Nsgb-open set K where u ϵ K ⊆ K. Then K is Nsgb-nbhd of u and for all points in K. Thus K is a Nsgb-nbhd of each of its points where K is Nsgb-open.

Sufficient Part: Let K be Nsgb-nbhd for each of its points. If K = ∅, it is Nsgb-open. If K ≠ ∅, then for each u ϵ K, ∃ a Nsgb-open set K_u such that u ϵ K_u ⊆ K. Then K = ∪ {K_u: u ϵ K}. But each K_u being Nsgb-open set and the arbitrary union of Nsgb-open is Nsgb-open. Therefore K is Nsgb-open.

Theorem 3.8: Axioms/ Properties of Nsgb-neighbourhood:

In ($\mathcal{U}$, $\tau_{\mathcal{R}}$(S)) we have

i) every u ϵ $\mathcal{U}$ has at least one Nsgb-nbhd.
ii) every Nsgb-nbhd of u ϵ $\mathcal{U}$ contains u.
iii) every superset of a Nsgb-nbhd of u ϵ $\mathcal{U}$ is a Nsgb-nbhd of u. if N ϵ N_u, N ⊂ M ⇒ M ϵ N_u
iv) if N is a Nsgb-nbhd of a point u ϵ $\mathcal{U}$, then ∃ a Nsgb-nbhd M of u with M ⊆ N then M is a Nsgb-nbhd of each of its points (and also M is Nsgb-open).

Proof:

i) Since $\mathcal{U}$ being a Nsgb-open set is is a Nsgb-nbhd of each of its points. So each u ϵ $\mathcal{U}$ has atleast one Nsgb-nbhd namely $\mathcal{U}$.
ii) Let N be Nsgb-nbhd of a point u ϵ $\mathcal{U}$. Then ∃ Nsgb-open set H where u ϵ H ⊆ N. Clearly u ϵ N, for all N. Thus each Nsgb-bhd of u contains u.
iii) Let N be a Nsgb-nbhd of u ϵ $\mathcal{U}$ and take M be a superset of N. N ⊂ M. Then by definition of Nsgb-nbhd, ∃ a Nsgb-open set H where u ϵ H ⊆ N ⊂ M implies u ϵ H ⊂ M. Then M is Nsgb-nbhd of u. Hence every superset of Nsgb-nbhd is a Nsgb-nbhd.
iv) Let N be Nsgb-nbhd of u ϵ $\mathcal{U}$. Then ∃ a Nsgb-open set M where u ϵ M ⊆ N. As M is Nsgb-open, it is Nsgb-nbhd of each of its points. [M itself is a Nsgb-nbhd of u and also Nsgb-nbhd of all points in M].

Theorem 3.9: Let F be Nsgb-closed subset in ($\mathcal{U}$, $\tau_{\mathcal{R}}$(S)). Let u ϵ F^C then there is a Nsgb-nbhd N of u such that N ∩ F = ∅.

Proof: Let F ⊂ $\mathcal{U}$ be Nsgb-closed and let u ϵ F^C. Since F is Nsgb-closed, F^C is Nsgb-open and it is a Nsgb-nbhd of each of its points. So we can say u ϵ F^C ⊆ F^C. Set F^C = N. Then u ϵ F^C ⊆ N implies N is Nsgb-nbhd of u. Thus F ∩ N = F ∩ F^C = ∅. Hence ∃ a Nsgb-nbhd N of u such that N ∩ F = ∅.

Theorem 3.10: Arbitrary union of Nsgb-neighbourhoods of a point u of $\mathcal{U}$ is Nsgb-neighbourhood of u.

Proof: Let {N_i}, i ϵ I (I denotes the index set) be an arbitrary collectionof Nsgb-nbhds of a point u ϵ $\mathcal{U}$. *To prove:* $\bigcup_{i\in I} N_i$ *is Nsgb-nbhd of u.* Since N_i is a Nsgb-nbhd of u, ∃ a Nsgb-open set H_i where u ϵ H_i ⊆ N_i, for all i ϵ I. Since N_i ⊆ ∪ N_i, u ϵ H_i ⊆ N_i ⊆ ∪ N_i, ∀ i ϵ I implies u ϵ H_i ⊆ ∪ N_i, ∀ i ϵ I. Thus $\bigcup_{i\in I} N_i$ is a Nsgb-nbhd of u.

Remark: Intersection of Nsgb-nbhd of a point need not be Nsgb-nbhd of that point.

Example 3.11: From example 3.3, let take a point k ϵ $\mathcal{U}$. Then Nsgb-$N_{(k)}$ = {{i, k}, {j, k}, {i, j, k}, {j, k, l}, {i, k, l}, $\mathcal{U}$} ⇒ ∩ Nsgb-$N_{(k)}$ = {k} ∉ Nsgb-$N_{(k)}$. ie)., {k} is not a Nsgb-nbhd of k.

Theorem 3.12: Every nano-nbhd of a point is Nsgb-nbhd of that point.

Proof: Let N be any nano-nbhd of u ϵ $\mathcal{U}$. Then ∃ a nano open set H where u ϵ H ⊆ N. Since each nano open is Nsgb-open, H is Nsgb-open and u ϵ H ⊆ N. Thus by definition of Nsgb-nbhd, N is Nsgb-nbhd of u.

Conversely, as in example 3.3: Take an element k of $\mathcal{U}$, then its nano nbhd system of k, Nano $N_{(k)}$ = {$\mathcal{U}$}, Nsgb-nbhd system of k, Nsgb-$N_{(k)}$ = {{i, k}, {i, j, k}, {i, k, l}, {j, k}, {j, k, l}, $\mathcal{U}$}. Here k ϵ {i, k} ⊆ {i, k, l} implies {i, k, l} is a Nsgb-nbhd of k but it is not a nano nbhd of k, since there exist no nano open set containing k contained in {i, k, l}.

Theorem 3.13: Let the nano topological spaces be ($\mathcal{U}$, $\tau_{\mathcal{R}}$(S)), ($\mathcal{V}$, $\tau_{\mathcal{R}}$(T)) and d: $\mathcal{U} \to \mathcal{V}$. Then d is Nsgb-continuous iff for each u ϵ $\mathcal{U}$, the inverse image under d of each nano nbhd of d(u) in $\mathcal{V}$ is a Nsgb-nbhd of u in $\mathcal{U}$.

Proof:

Necessary Part: Take d: $\mathcal{U} \rightarrow \mathcal{V}$ be Nsgb-continuous. Let u ϵ $\mathcal{U}$ then d(u) ϵ $\mathcal{V}$. Let M be any nano-nbhd of d(u) in $\mathcal{V}$ then there is a nano open set H suth that d(u) ϵ H ⊆ M implies u ϵ d^{-1}(H) ⊆ d^{-1}(M). As d is Nsgb-continuous, d^{-1}(H) is Nsgb-open in $\mathcal{U}$ then d^{-1}(M) is Nsgb-nbhd of u in $\mathcal{U}$.

Sufficient Part: Consider that the inverse image of every nano-nbhd of d(u) in $\mathcal{V}$ is a Nsgb-nbhd of u in $\mathcal{U}$. Let H be any nano open set in $\mathcal{V}$. Suppose d^{-1}(H) = ∅, d^{-1}(H) is Nsgb-open. If d^{-1}(H) ≠ ∅ and u ϵ d^{-1}(H) in $\mathcal{U}$, then u ϵ d^{-1}(H) ⊆ $\mathcal{U}$ implies d(u) ϵ H in $\mathcal{V}$ that is d(u) ϵ H ⊆ $\mathcal{V}$. This shows H is a nano nbhd of d(u) in $\mathcal{V}$. By the given hypothesis, d^{-1}(H) is Nsgb-nbhd of u in $\mathcal{U}$ and it is Nsgb-nbhd for each of its points. This implies d^{-1}(H) is Nsgb-open in $\mathcal{U}$. Thus d is Nsgb-continuous.

Nsgb-Adherent Points and Nsgb-Derived Sets

Definition 4.1: In ($\mathcal{U}$, $\tau_{\mathcal{R}}$(S)), let A ⊆ $\mathcal{U}$. A point u ϵ $\mathcal{U}$ is termed as **Nsgb-adherent point** or **Nsgb-contact point of A**, if every Nsgb-open set containing u, contains atleast one point of A. Nsgb-adherent points are of two types: 1. Nsgb-limit point 2. Nsgb-isolated point.

Definition 4.2: Let A ⊆ $\mathcal{U}$. A point u ∈ $\mathcal{U}$ is called **Nsgb-limit point** or **Nsgb-accumulation point** or **Nsgb-cluster point** iff every Nsgb-open set containing u, contains atleast one point of A other than u.
ie)., u is a Nsgb-limit point of A ⇔ (H – {u}) ∩ A ≠ ∅, for all Nsgb-open set H containing u.

The set of all Nsgb-limit points of A is the **Nsgb-derived set of A** and expressed by Nsgb-D(A).

Definition 4.3:

- A Nsgb-adherent point of A that is not a Nsgb-limit point of A is said to be a **Nsgb-isolated point of A**.
- If all the points of A is Nsgb-isolated point, then A is **Nsgb-isolated set.**
- A set A is **Nsgb-perfect set** if A is Nsgb-closed and it has no Nsgb-isolated point.

Remark:

- An Nsgb-adherent point is either an Nsgb-isolated point or a Nsgb-limit point of A.
  ie)., Nsgb-Adh(A) = A ∪ Nsgb-D(A).
- For if u is a Nsgb-adherent point of A, then there are the following two mutually exclusive possibilities.

Case 1: Every Nsgb-open set containing u, has a point of A other than u, then u is a Nsgb-limit point of A.
Case 2: u ∈ A and there is some Nsgb-open set containing u, which contains no point of A except u, then u is a Nsgb-isolated point.

Example 4.4: Take $\mathcal{U}$ = {i, j, k, l}, S = {k, l}, $\mathcal{U}/\mathcal{R}$ = {{i}, {k}, {j, l}}, $\tau_{\mathcal{R}}$(S) = {$\mathcal{U}$, ∅, {k}, {j, k, l}, {j, l}} Nsgb-open = {$\mathcal{U}$, ∅, {j}, {k}, {l}, {i, k}, {j, k}, {j, l}, {k, l}, {i, j, k}, {i, k, l}, {j, k, l}}

1. Let A = {j, k} ⊂ $\mathcal{U}$. Here j, k are not Nsgb-limit point for {j, k}. Because {j} and {k} are the Nsgb-open sets which does not contain any other point of A.

ie)., ({j} - j) ∩ {j, k} = ∅ ∩ {j, k} = ∅ and ({k} - k) ∩ {j, k} = ∅ ∩ {j, k} = ∅.
Is i Nsgb-limit point of A = {j, k}?

For i ∈ $\mathcal{U}$, Nsgb-open sets contain i are {i, k}, {i, j, k}, {i, k, l}, $\mathcal{U}$. Here every Nsgb-open sets containing i has atleast one point of A other than i. Hence i is a Nsgb-limit point of {j, k}.
And for l ∈ $\mathcal{U}$ is not a Nsgb-limit point for {j, k}. Since the Nsgb-open set {l} does not contain any point of A. Thus l is not a Nsgb-limit point.

Hence i is a Nsgb-limit point and j, k, l are Nsgb-isolated points of {j, k}. Also the set {j, k} is a Nsgb-isolated set since every point of A is Nsgb-isolated. And Nsgb-derived set of {j, k} = {i}. Nsgb-Adherence A = {i, j, k, l}.

2. Let B = {i, j, l}. B has no Nsgb-limit points. ⇒ Nsgb-D(B) = ∅.

For i ∈ $\mathcal{U}$, a Nsgb-open set {i, k} does not contain any other point of B. Similarly for j, k, l of $\mathcal{U}$, Nsgb-open sets {j}, {k}, {l} does not contain any other point of B. Hence i, j, k, l are not Nsgb-limit points of B so they are Nsgb-isolated points of B. Here the set B is a Nsgb-isolated set.

Theorem 4.5: (Characterization of Nsgb-closed set in terms of Nsgb-derived set)

Let K ⊆ $\mathcal{U}$ in ($\mathcal{U}$, $\tau_{\mathcal{R}}$(S)). Then K is Nsgb-closed iff Nsgb-D(K) ⊆ K.

Proof: ***Necessary Part:*** Assume K be Nsgb-closed. Then K^C is Nsgb-open and it is a Nsgb-neighbourhood of each of its points. Then for each u ∈ K^C, ∃ a Nsgb-open set N_u of u with N_u ⊆ K^C. Since K ∩ K^C = ϕ, the Nsgb-open set N_u does not contain any point of K then u is not a Nsgb-limit point of K. Hence no point of K^C is a Nsgb-limit point of K. Thus K contains all its Nsgb-limit points. Therefore Nsgb-D(K) ⊆ K.

Sufficient Part: Assume Nsgb-D(K) ⊆ K and take u ∈ K^C. So u ∉ K. Also u ∉ Nsgb-D(K) so that u is not a Nsgb-limit point of K. This implies that ∃ a Nsgb-open set N_u of u with N_u ∩ K = ϕ so that N_u ⊆ K^C. ie)., u ∈ N_u ⊆ K^C. Thus K^C is the Nsgb-neighbourhood for each of its points. Hence K^C is Nsgb-open and therefore K is Nsgb-closed.

Theorem 4.6: In a nano topological space, every Nsgb-closed subset is the union of its set of Nsgb-isolated points and set of Nsgb-limit points which are disjoint. (ie)., Nsgb-closed subset = Nsgb-isolated set ∪ Nsgb-derived set.

Proof: Let K be any subset. To prove K ∪ Nsgb-D(K) is a Nsgb-closed. We are going to prove $(K \cup \text{Nsgb-D}(K))^C$ is a Nsgb-open. If $(K \cup \text{Nsgb-D}(K))^C$ is ϕ, it is clearly Nsgb-open. If not, let u ∈ $(K \cup \text{Nsgb-D}(K))^C$ then u ∉ $(K \cup \text{Nsgb-D}(K))^C$ implies u ∉ K and u ∉ Nsgb-D(K). For u ∉ K, ∃ a Nsgb-open set H where u ∈ H and H ∩ K = ϕ gives u ∈ H ⊆ K^C → (1) For u ∉ Nsgb-D(K), again H being a Nsgb-open set containing no point of K. That is no point of H is a Nsgb-limit point of K implies u ∈ H ⊆ $(\text{Nsgb-D}(K))^C$ →(2). From (1) and (2), u ∈ H ⊆ K^C ∩ $(\text{Nsgb-D}(K))^C$ = $(K \cup \text{Nsgb-D}(K))^C$. Then $(K \cup \text{Nsgb-D}(K))^C$ is a Nsgb-nbhd of each of its points so that is Nsgb-open. Hence K ∪ Nsgb-D(K) is Nsgb-closed.

Theorem 4.7: (Properties of Nsgb-derived set)

Let K, L $\subseteq$ ($\mathcal{U}$, $\tau_{\mathcal{R}}$(S)) then

i) Nsgb-D(ϕ) = ϕ
ii) K $\subseteq$ L $\Rightarrow$ Nsgb-D(K) $\subseteq$ Nsgb-D(L)
iii) Nsgb-D(K∩L) $\subseteq$ Nsgb-D(K) ∩ Nsgb-D(L)
iv) Nsgb-D(K∪L) $\subseteq$ Nsgb-D(K) ∪ Nsgb-D(L)

Proof:

i. Since ϕ is Nsgb-closed and by previous theorem, Nsgb-D(ϕ) $\subseteq$ ϕ . But $\phi \subseteq$ Nsgb-D(ϕ). Hence Nsgb-D(ϕ) = ϕ.
ii. Let u ∈ Nsgb-D(K) then u is a Nsgb-limit point of K implies every Nsgb-open set containing u has a point of K other than u. Since K $\subseteq$ L, every Nsgb-open set containing u has a point of L other than u says u is Nsgb-limit point of L and so u ∈ Nsgb-D(L). Thus K $\subseteq$ L implies Nsgb-D(K) $\subseteq$ Nsgb-D(L).
iii. Since K∩L $\subseteq$ K and K∩L $\subseteq$ L, by (ii), Nsgb-D(K∩L) $\subseteq$ Nsgb-D(K) and Nsgb-D(K∩L) $\subseteq$ Nsgb-D(L) gives Nsgb-D(K∩L) $\subseteq$ Nsgb-D(K) ∩ Nsgb-D(L).
iv. Since K $\subseteq$ K∪L and L $\subseteq$ K∪L, by (ii), Nsgb-D(K) $\subseteq$ Nsgb-D(K∪L) and Nsgb-D(L) $\subseteq$ Nsgb-D(K∪L) gives Nsgb-D(K) ∪ Nsgb-D(L) $\subseteq$ Nsgb-D(K∪L).

Application of nano topology – Finding root cause for Hair fall

Consider $\mathcal{U}$ as the universe set, $\mathcal{A}$ as the attributes set. The value set V_a can be associated to each of its attribute a in $\mathcal{A}$. An intriguing question concerns if the information system has a particular set of attributes that by themselves may fully characterize the data stored in the database. There might be several reducts in the information system. In particular, however, we have an affinity for a specific reduct, which could be the minimal reduct or any reduct that contains the key attributes. Such a collection of reduct attributes is the CORE based on the basis.

Definition 5.1: Assume ($\mathcal{U}$, $\mathcal{A}$) be the system of information with conditional attributes $\mathcal{C}$ and decision attribute $\mathcal{D}$ of $\mathcal{A}$. The **Core** is a subset $\mathcal{E}$ of $\mathcal{C}$, if $\mathcal{B}_{\mathcal{E}} = \mathcal{B}_{\mathcal{C}}$ and $\mathcal{B}_{\mathcal{E}} \neq \mathcal{B}_{\mathcal{E} - \{e\}}$ for every e ∈ $\mathcal{E}$, whereas $\mathcal{B}_{\mathcal{E}}$, the basis of nano topology pertaining to $\mathcal{E} \subseteq \mathcal{C}$.

ie)., a core comprises a minimal number of attributes which cannot be deleted without impacting the categorization effectiveness of the attributes.

Algorithm:

Step 1: Having a finite universe $\mathcal{U}$, a finite attribute set $\mathcal{A}$ and subdivided into two groups, condition attributes $\mathcal{C}$ and decision attribute $\mathcal{D}$, an equivalence class relation $\mathcal{R}$ on $\mathcal{U}$ pertaining to $\mathcal{C}$ and G a subset in $\mathcal{U}$ express the data as in information table, each column have a attribute and row have objects. The entries expressed the attribute values.
Step 2: Obtain the lower, upper approximations and boundary region in relation to $\mathcal{C}$.
Step 3: Determine nano topology $\tau_{\mathcal{R}(\mathcal{C})}$(G) on $\mathcal{U}$, its basis $\mathcal{B}_{(\mathcal{C})}$(G) pertaining to set $\mathcal{C}$.
Step 4: Eliminate an attribute g form $\mathcal{C}$ and obtain lower, upper approximations and boundary region for G regarding the equivalence relation $\mathcal{U}/\mathcal{R}_{(\mathcal{C}-\{g\})}$.
Step 5: Determine the nano topology $\tau_{\mathcal{R}(\mathcal{C}-\{g\})}$ (G) on $\mathcal{U}$, its basis $\mathcal{B}_{(\mathcal{C}-\{g\})}$ (G).

Step 6: Step 3 along with 4 should be followed for every attribute in $\mathcal{C}$.

Step 7: The attribute elements in $\mathcal{C}$ with $\mathcal{B}_{(C-\{g\})}(G) \neq \mathcal{B}_{(C)}(G)$ constitute the **CORE**.

Problem Statement:

Now-a-days hair fall/ hair loss is common for all due to many reasons like nutrient deficiency, water (TDS level), medical conditions of individuals, frequent hair styling using chemical products, pollution, stress and so on.

The problem is to identify the root causes of hair fall among people in groups using topological attribute reduction in the complete information system based on the basis of nano topology.

Here the given table 1 represents the people in groups with/ without hair fall under the circumstances.

Table 1. Groups of people with or without hairfall under the circumstances

GROUPS	DIET (D)	MEDICATIONS (M)	WATER (W)	HAIR STYLING (H)	STRESS (S)	OBSERVATION (O) (Fall/ No fall)
Alpha(α)	Poor	No	Moderate	No	No	Fall
Eta(η)	Good	No	Heavy	No	No	Fall
Delta(δ)	Good	No	Good	No	No	No Fall
Beta(β)	Average	No	Moderate	Yes	No	No Fall
Zeta(ζ)	Good	Yes	Good	No	Yes	Fall
Gamma(γ)	Poor	No	Good	No	No	Fall
Theta(θ)	Average	No	Moderate	No	No	No Fall
Lamda(λ)	Average	No	Moderate	Yes	Yes	Fall

The groups formed accordingly the data given by the individuals.

Here $\mathcal{U} = \{\alpha, \eta, \delta, \beta, \zeta, \gamma, \theta, \lambda\}$ constitute the groups of individuals is the universe set, $\mathcal{A}$ = {Diet, Medications, Water, Hair styling, Stress, Observation} represent the set of attributes both conditions and decision.

Columns indicate the reasons of hair fall, rows indicate the groups, entries in the table indicate the attribute values. {Diet, Medications, Water, Hair styling, Stress} are the conditional attributes and {Observation (Fall/ No fall)} is the decision attribute.

The table values denoted as $\mathcal{U} = \{\alpha, \eta, \delta, \beta, \zeta, \gamma, \theta, \lambda\}$, $\mathcal{A} = \{D, M, W, S, H, O\}$, $\mathcal{C} = \{D, M, W, H, S\} \subset \mathcal{A}$, $\mathcal{D} = \{O\} \subset \mathcal{A}$.
The group ζ can be characterized by the value set as ζ = {D, Good}, {M, Yes}, {W, Good}, {H, No}, {S, Yes} and {O, Fall}

The equivalence classes generated by the conditional attributes are given by

$\mathcal{U}/\mathcal{R}_{(D)} = \{\{\alpha, \gamma\}, \{\beta, \theta, \lambda\}, \{\eta, \delta, \zeta\}\}$, $\mathcal{U}/\mathcal{R}_{(M)} = \{\{\alpha, \eta, \delta, \beta, \gamma, \theta, \lambda\}, \{\zeta\}\}$, $\mathcal{U}/\mathcal{R}_{(W)} = \{\{\alpha, \beta, \theta, \lambda\}, \{\eta\}, \{\delta, \zeta, \gamma\}\}$, $\mathcal{U}/\mathcal{R}_{(H)} = \{\{\alpha, \eta, \delta, \zeta, \gamma, \theta\}, \{\beta, \lambda\}\}$, $\mathcal{U}/\mathcal{R}_{(S)} = \{\{\alpha, \eta, \delta, \beta, \gamma, \theta\}, \{\zeta, \lambda\}\}$

The equivalence class in respect of all the conditions $\mathcal{U}/\mathcal{R}_{(C)}$ = {{α}, {η}, {δ}, {β}, {ζ}, {γ}, {θ}, {λ}}.

Before decision rules might be formed, not all condition attributes in a system of information must describe the decision attribute. i.e., it could occur that the decision attribute is dependent on a subset of the condition attributes rather than the entire set. As a result, to figure out this subset, which is referred as the core.

CASE 1: Groups having Hair Fall

Let the group having hair fall be Σ = {α, η, ζ, γ, λ}. Let an equivalence relation of $\mathcal{U}$ be $\mathcal{R}$ with respect to all conditional attributes C. $\mathcal{U}/\mathcal{R}_{(C)}(\Sigma)$ = {{α}, {η}, {δ}, {β}, {ζ}, {γ}, {θ}, {λ}}. Then with respect to C, Lower approximation = $L_{\mathcal{R}(C)}(\Sigma)$ = {α, η, ζ, γ, λ}, Upper approximation = $U_{\mathcal{R}(C)}(\Sigma)$ = {α, η, ζ, γ, λ}, Boundary region with respect to C = $B_{\mathcal{R}(C)}(\Sigma)$ = ϕ. Then nano topology on $\mathcal{U}$ by C, $\tau_{\mathcal{R}(C)}(\Sigma)$ = {$\mathcal{U}$, ϕ, {α, η, ζ, γ, λ}} and basis of $\tau_{\mathcal{R}(C)}(\Sigma)$ = $\mathcal{B}_{(C)}(\Sigma)$ = {$\mathcal{U}$, {α, η, ζ, γ, λ}, ϕ}

Step 1:

i) Eliminate an attribute **'Diet'** from C.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(C\text{-}D)}(\Sigma)$ = {{α, θ}, {η}, {ζ}, {δ, γ}, {β}, {λ}} and the pertaining $L_{\mathcal{R}(C\text{-}D)}(\Sigma)$ = {η, ζ, λ}, $U_{\mathcal{R}(C\text{-}D)}(\Sigma)$ = {α, η, δ, ζ, γ, θ, λ}, $B_{\mathcal{R}(C\text{-}D)}(\Sigma)$ = {α, δ, γ, θ} then nano topology on $\mathcal{U}$ by C-D, $\tau_{\mathcal{R}(C\text{-}D)}(\Sigma)$ = {$\mathcal{U}$, ϕ, {η, ζ, λ}, {α, η, δ, ζ, γ, θ, λ}, { α, δ, γ, θ}} and its basis $\mathcal{B}_{(C\text{-}D)}(\Sigma)$ = {$\mathcal{U}$, ϕ, {η, ζ, λ}, {α, δ, γ, θ}} ≠ $\mathcal{B}_{(C)}(\Sigma)$

ii) Eliminate an attribute **'Medications'** from C.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(C - M)}(\Sigma)$ = {{α}, {η}, {δ}, {β}, {ζ}, {γ}, {θ}, {λ}} and the pertaining $L_{\mathcal{R}(C - M)}(\Sigma)$ = {α, η, ζ, γ, λ}, $U_{\mathcal{R}(C - M)}(\Sigma)$ = {α, η, ζ, γ, λ}, $B_{\mathcal{R}(C - M)}(\Sigma)$ = ϕ then nano topology on $\mathcal{U}$ by C-M, $\tau_{\mathcal{R}(C - M)}(\Sigma)$ = {$\mathcal{U}$, ϕ, {α, η, ζ, γ, λ}} and its basis $\mathcal{B}_{(C - M)}(\Sigma)$ = {$\mathcal{U}$, {α, η, ζ, γ, λ}, ϕ} = $\mathcal{B}_{(C)}(\Sigma)$

iii) Eliminate an attribute **'Water'** from C.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(C - W)}(\Sigma)$ = {{α, γ}, {ζ}, {η, δ}, {β}, {θ}, {λ}} and the pertaining $L_{\mathcal{R}(C - W)}(\Sigma)$ = {α, γ, ζ, λ}, $U_{\mathcal{R}(C - W)}(\Sigma)$ = {α, γ, ζ, η, δ, λ}, $B_{\mathcal{R}(C - W)}(\Sigma)$ = {η, δ} then nano topology on $\mathcal{U}$ by C - W, $\tau_{\mathcal{R}(C - W)}(\Sigma)$ = {$\mathcal{U}$, ϕ, {α, γ, ζ, λ}, {α, γ, ζ, η, δ, λ}, {η, δ}} and its basis $\mathcal{B}_{(C - W)}(\Sigma)$ = {$\mathcal{U}$, ϕ, {α, ζ, γ, λ}, {η, δ}} ≠ $\mathcal{B}_{(C)}(\Sigma)$

iv) Eliminate an attribute **'Hair Styling'** from C.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(C - H)}(\Sigma)$ = {{α}, {η}, {δ}, {β, θ}, {ζ}, {γ}, {λ}} and the pertaining $L_{\mathcal{R}(C - H)}(\Sigma)$ = {α, η, ζ, γ, λ}, $U_{\mathcal{R}(C - H)}(\Sigma)$ = {α, η, ζ, γ, λ}, $B_{\mathcal{R}(C - H)}(\Sigma)$ = ϕ then nano topology on $\mathcal{U}$ by C - H, $\tau_{\mathcal{R}(C - H)}(\Sigma)$ = {$\mathcal{U}$, ϕ, {α, η, ζ, γ, λ}} and its basis $\mathcal{B}_{(C - H)}(\Sigma)$ = {$\mathcal{U}$, {α, η, ζ, γ, λ}, ϕ} = $\mathcal{B}_{(C)}(\Sigma)$

v) Eliminate an attribute **'Stress'** from $\mathcal{C}$.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(\mathcal{C}\text{ - S})}(\Sigma) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta, \lambda\}, \{\zeta\}, \{\gamma\}, \{\theta\}\}$ and the pertaining $L_{\mathcal{R}(\mathcal{C}\text{ - S})}(\Sigma) = \{\alpha, \eta, \zeta, \gamma\}$, $U_{\mathcal{R}(\mathcal{C}\text{ - S})}(\Sigma) = \{\alpha, \eta, \beta, \lambda, \zeta, \gamma\}$, $B_{\mathcal{R}(\mathcal{C}\text{ - S})}(\Sigma) = \{\beta\}$ then nano topology on $\mathcal{U}$ by $\mathcal{C}$ - S, $\tau_{\mathcal{R}(\mathcal{C}\text{ - S})}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \eta, \zeta, \gamma\}, \{\alpha, \eta, \beta, \lambda, \zeta, \gamma\}, \{\beta\}\}$ and its basis $\mathcal{B}_{(\mathcal{C}\text{ - S})}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \eta, \zeta, \gamma\}, \{\beta\}\} \neq \mathcal{B}_{(\mathcal{C})}(\Sigma)$

STEP 2(A):

Let $G_1 = \mathcal{C} - M = \{D, W, H, S\}$ and $\mathcal{U}/\mathcal{R}_{(G1)}(\Sigma) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta\}, \{\zeta\}, \{\gamma\}, \{\theta\}, \{\lambda\}\}$ and its basis $B_{(G1)}(\Sigma) = \{\mathcal{U}, \{\alpha, \eta, \zeta, \gamma, \lambda\}, \phi\} = B_{(\mathcal{C})}(\Sigma)$

i) Eliminate an attribute **'Diet'** from G_1

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(G1\text{ - D})}(\Sigma) = \{\{\alpha, \theta\}, \{\delta, \gamma\}, \{\eta\}, \{\beta\}, \{\zeta\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(G1\text{ - D})}(\Sigma) = \{\eta, \zeta, \lambda\}$, $U_{\mathcal{R}(G1\text{ - D})}(\Sigma) = \{\alpha, \theta, \delta, \gamma, \zeta, \eta, \lambda\}$, $B_{\mathcal{R}(G1\text{ - D})}(\Sigma) = \{\alpha, \delta, \gamma, \theta\}$ then nano topology on $\mathcal{U}$ by G_1 - D, $\tau_{\mathcal{R}(G1\text{ - D})}(\Sigma) = \{\mathcal{U}, \phi, \{\eta, \zeta, \lambda\}, \{\alpha, \eta, \delta, \zeta, \gamma, \theta, \lambda\}, \{\alpha, \delta, \gamma, \theta\}\}$ and its basis $\mathcal{B}_{(G1\text{ - D})}(\Sigma) = \{\mathcal{U}, \phi, \{\eta, \zeta, \lambda\}, \{\alpha, \delta, \gamma, \theta\}\} \neq \mathcal{B}_{(G1)}(\Sigma)$

ii) Eliminate an attribute **'Water'** from G_1.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(G1\text{ - W})}(\Sigma) = \{\{\alpha, \gamma\}, \{\eta, \delta\}, \{\beta\}, \{\zeta\}, \{\theta\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(G1\text{ - W})}(\Sigma) = \{\alpha, \zeta, \gamma, \lambda\}$, $U_{\mathcal{R}(G1\text{ - W})}(\Sigma) = \{\alpha, \eta, \delta, \zeta, \gamma, \lambda\}$, $B_{\mathcal{R}(G1\text{ - W})}(\Sigma) = \{\eta, \delta\}$ then nano topology on $\mathcal{U}$ by G_1 - W, $\tau_{\mathcal{R}(G1\text{ - W})}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \zeta, \gamma, \lambda\}, \{\alpha, \eta, \delta, \zeta, \gamma, \lambda\}, \{\eta, \delta\}\}$ and its basis $\mathcal{B}_{(G1\text{ - W})}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \zeta, \gamma, \lambda\}, \{\eta, \delta\}\} \neq \mathcal{B}_{(G1)}(\Sigma)$

iii) Eliminate an attribute **'Hair Styling'** from G_1.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(G1\text{ - H})}(\Sigma) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta, \theta\}, \{\zeta\}, \{\gamma\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(G1\text{ - H})}(\Sigma) = \{\alpha, \eta, \zeta, \gamma, \lambda\}$, $U_{\mathcal{R}(G1\text{ - H})}(\Sigma) = \{\alpha, \eta, \zeta, \gamma, \lambda\}$, $B_{\mathcal{R}(G1\text{ - H})}(\Sigma) = \phi$ then nano topology on $\mathcal{U}$ by G_1 - H, $\tau_{\mathcal{R}(G1\text{ - H})}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \eta, \zeta, \gamma, \lambda\}\}$ and its basis $\mathcal{B}_{(G1\text{ - H})}(\Sigma) = \{\mathcal{U}, \{\alpha, \eta, \zeta, \gamma, \lambda\}, \phi\} = \mathcal{B}_{(G1)}(\Sigma)$

iv) Eliminate an attribute **'Stress'** from G_1.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(G1\text{ - S})}(\Sigma) = \{\{\alpha\}, \{\eta\}, \{\delta, \zeta\}, \{\beta, \lambda\}, \{\gamma\}, \{\theta\}\}$ and the pertaining $L_{\mathcal{R}(G1\text{ - S})}(\Sigma) = \{\alpha, \eta, \gamma\}$, $U_{\mathcal{R}(G1\text{ - S})}(\Sigma) = \{\alpha, \eta, \delta, \beta, \zeta, \gamma, \lambda\}$, $B_{\mathcal{R}(G1\text{ - S})}(\Sigma) = \{\delta, \beta, \zeta, \lambda\}$ then nano topology on $\mathcal{U}$ by G_1 - S, $\tau_{\mathcal{R}(G1\text{ - S})}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \eta, \gamma\}, \{\alpha, \eta, \delta, \beta, \zeta, \gamma, \lambda\}, \{\delta, \beta, \zeta, \lambda\}\}$ and its basis $\mathcal{B}_{(G1\text{ - S})}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \eta, \gamma\}, \{\delta, \beta, \zeta, \lambda\}\} \neq \mathcal{B}_{(G1)}(\Sigma)$

STEP 2(B):

Let $G_2 = \mathcal{C} - H = \{D, M, W, S\}$ and $\mathcal{U}/\mathcal{R}_{(G2)}(\Sigma) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta, \theta\}, \{\zeta\}, \{\gamma\}, \{\lambda\}\}$ and its basis $\mathcal{B}_{(G2)}(\Sigma) = \{\mathcal{U}, \{\alpha, \eta, \zeta, \gamma, \lambda\}, \phi\} = \mathcal{B}_{(\mathcal{C})}(\Sigma)$

i) Eliminate an attribute **'Diet'** from G_2.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(G2 - D)}(\Sigma) = \{\{\alpha, \beta, \theta\}, \{\eta\}, \{\delta, \gamma\}, \{\zeta\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(G2 - D)}(\Sigma) = \{\eta, \zeta, \lambda\}$, $U_{\mathcal{R}(G2 - D)}(\Sigma) = \{\alpha, \beta, \theta, \eta, \delta, \gamma, \zeta, \lambda\}$, $B_{\mathcal{R}(G2 - D)}(\Sigma) = \{\alpha, \beta, \theta, \delta, \gamma\}$ then nano topology on $\mathcal{U}$ by G_2 - D, $\tau_{\mathcal{R}(G2 - D)}(\Sigma) = \{\mathcal{U}, \phi, \{\eta, \zeta, \lambda\}, \{\alpha, \eta, \delta, \beta, \zeta, \gamma, \theta, \lambda\}, \{\alpha, \delta, \beta, \gamma, \theta\}\}$ and its basis $\mathcal{B}_{(G2 - D)}(\Sigma) = \{\mathcal{U}, \phi, \{\eta, \zeta, \lambda\}, \{\alpha, \delta, \beta, \gamma, \theta\}\} \neq \mathcal{B}_{(G2)}(\Sigma)$

ii) Eliminate an attribute **'Medications'** from G_2.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(G2 - M)}(\Sigma) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta, \theta\}, \{\zeta\}, \{\gamma\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(G2 - M)}(\Sigma) = \{\alpha, \eta, \zeta, \gamma, \lambda\}$, $U_{\mathcal{R}(G2 - M)}(\Sigma) = \{\alpha, \eta, \zeta, \gamma, \lambda\}$, $B_{\mathcal{R}(G2 - M)}(\Sigma) = \phi$ then nano topology on $\mathcal{U}$ by G_2 - M, $\tau_{\mathcal{R}(G2 - M)}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \eta, \zeta, \gamma, \lambda\}\}$ and its basis $\mathcal{B}_{(G2 - M)}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \eta, \zeta, \gamma, \lambda\}\} = \mathcal{B}_{(G2)}(\Sigma)$

iii) Eliminate an attribute **'Water'** from G_2.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(G2 - W)}(\Sigma) = \{\{\alpha, \gamma\}, \{\eta, \delta\}, \{\beta, \theta\}, \{\zeta\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(G2 - W)}(\Sigma) = \{\alpha, \gamma, \zeta, \lambda\}$, $U_{\mathcal{R}(G2 - W)}(\Sigma) = \{\alpha, \gamma, \eta, \delta, \zeta, \lambda\}$, $B_{\mathcal{R}(G2 - W)}(\Sigma) = \{\eta, \delta\}$ then nano topology on $\mathcal{U}$ by G_2 - W, $\tau_{\mathcal{R}(G2 - W)}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \zeta, \gamma, \lambda\}, \{\alpha, \eta, \delta, \zeta, \gamma, \lambda\}, \{\eta, \delta\}\}$ and its basis $\mathcal{B}_{(G2 - W)}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \zeta, \gamma, \lambda\}, \{\eta, \delta\}\} \neq \mathcal{B}_{(G2)}(\Sigma)$

iv) Eliminate an attribute **'Stress'** from G_2.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(G2 - S)}(\Sigma) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta, \theta, \lambda\}, \{\zeta\}, \{\gamma\}\}$ and the pertaining $L_{\mathcal{R}(G2 - S)}(\Sigma) = \{\alpha, \eta, \zeta, \gamma\}$, $U_{\mathcal{R}(G2 - S)}(\Sigma) = \{\alpha, \eta, \delta, \beta, \theta, \lambda, \zeta, \gamma\}$, $B_{\mathcal{R}(G2 - S)}(\Sigma) = \{\delta, \beta, \zeta, \lambda\}$ then nano topology on $\mathcal{U}$ by G_2 - S, $\tau_{\mathcal{R}(G2 - S)}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \eta, \zeta, \gamma\}, \{\delta, \beta, \theta, \lambda\}\}$ and its basis $\mathcal{B}_{(G2 - S)}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \eta, \zeta, \gamma\}, \{\delta, \beta, \theta, \lambda\}\} \neq \mathcal{B}_{(G2)}(\Sigma)$

STEP 3:

Let $Z = \mathcal{C} - M$, $H = \{D, W, S\}$ and $\mathcal{U}/\mathcal{R}_{(Z)}(\Sigma) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta, \theta\}, \{\zeta\}, \{\gamma\}, \{\lambda\}\}$ and its basis $\mathcal{B}_{(Z)}(\Sigma) = \{\mathcal{U}, \{\alpha, \eta, \zeta, \gamma, \lambda\}, \phi\} = \mathcal{B}_{(C)}(\Sigma)$

i) Eliminate an attribute **'Diet'** from Z.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(Z - D)}(\Sigma) = \{\{\alpha, \beta, \theta\}, \{\delta, \gamma\}, \{\alpha\}, \{\zeta\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(Z - D)}(\Sigma) = \{\eta, \zeta, \lambda\}$, $U_{\mathcal{R}(Z - D)}(\Sigma) = \{\alpha, \eta, \delta, \beta, \zeta, \gamma, \theta, \lambda\}$, $B_{\mathcal{R}(Z - D)}(\Sigma) = \{\alpha, \delta, \beta, \gamma, \theta\}$ then nano topology on $\mathcal{U}$ by Z - D, $\tau_{\mathcal{R}(Z - D)}(\Sigma) = \{\mathcal{U}, \phi, \{\eta, \zeta, \lambda\}, \{\alpha, \delta, \beta, \gamma, \theta\}\}$ and its basis $\mathcal{B}_{(Z - D)}(\Sigma) = \{\mathcal{U}, \phi, \{\eta, \zeta, \lambda\}, \{\alpha, \delta, \beta, \gamma, \theta\}\} \neq \mathcal{B}_{(Z)}(\Sigma)$

ii) Eliminate an attribute **'Water'** from Z.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(Z-W)}(\Sigma) = \{\{\alpha, \gamma\}, \{\eta, \delta\}, \{\beta, \theta\}, \{\zeta\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(Z-W)}(\Sigma) = \{\alpha, \gamma, \zeta, \lambda\}$, $U_{\mathcal{R}(Z-W)}(\Sigma) = \{\alpha, \gamma, \eta, \delta, \zeta, \lambda\}$, $B_{\mathcal{R}(Z-W)}(\Sigma) = \{\eta, \delta\}$ then nano topology on $\mathcal{U}$ by Z - W, $\tau_{\mathcal{R}(Z-W)}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \gamma, \zeta, \lambda\}, \{\alpha, \gamma, \eta, \delta, \zeta, \lambda\}, \{\eta, \delta\}\}$ and its basis $\mathcal{B}_{(Z-W)}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \gamma, \zeta, \lambda\}, \{\eta, \delta\}\} \neq \mathcal{B}_{(Z)}(\Sigma)$

iii) Eliminate an attribute **'Stress'** from Z.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(Z-S)}(\Sigma) = \{\{\alpha\}, \{\eta\}, \{\delta, \zeta\}, \{\beta, \theta, \lambda\}, \{\gamma\}\}$ and the pertaining $L_{\mathcal{R}(Z-S)}(\Sigma) = \{\alpha, \eta, \gamma\}$, $U_{\mathcal{R}(Z-S)}(\Sigma) = \{\alpha, \eta, \delta, \beta, \theta, \lambda, \zeta, \gamma\}$, $B_{\mathcal{R}(Z-S)}(\Sigma) = \{\delta, \beta, \zeta, \theta, \lambda\}$ then nano topology on $\mathcal{U}$ by Z - S, $\tau_{\mathcal{R}(Z-S)}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \eta, \gamma\}, \{\delta, \beta, \zeta, \theta, \lambda\}\}$ and its basis $\mathcal{B}_{(Z-S)}(\Sigma) = \{\mathcal{U}, \phi, \{\alpha, \eta, \gamma\}, \{\delta, \beta, \zeta, \theta, \lambda\}\} \neq \mathcal{B}_{(Z)}(\Sigma)$

Thus core = {Diet, Water, Stress}.

CASE 2: Groups does not have Hair Fall

Let the groups does not have hair fall be $\Psi = \{\delta, \beta, \theta\}$. Let an equivalence relation of $\mathcal{U}$ be $\mathcal{R}$ with respect to all conditional attributes C. $\mathcal{U}/\mathcal{R}_{(C)}(\Psi) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta\}, \{\zeta\}, \{\gamma\}, \{\theta\}, \{\lambda\}\}$. Then Lower approximation with respect to $C = L_{\mathcal{R}(C)}(\Psi) = \{\delta, \beta, \theta\}$, Upper approximation with respect to $C = U_{\mathcal{R}(C)}(\Psi) = \{\delta, \beta, \theta\}$, Boundary region with respect to $C = B_{\mathcal{R}(C)}(\Psi) = \phi$. Then nano topology on $\mathcal{U}$ by C, $\tau_{\mathcal{R}(C)}(\Psi) = \{\mathcal{U}, \phi, \{\delta, \beta, \theta\}\}$ and basis of $\tau_{\mathcal{R}(C)}(\Psi)$, $\mathcal{B}_{(C)}(\Psi) = \{\mathcal{U}, \{\delta, \beta, \theta\}, \phi\}$

Step 1:

i) Eliminate an attribute **'Diet'** from C.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(C-D)}(\Psi) = \{\{\alpha, \theta\}, \{\eta\}, \{\delta, \gamma\}, \{\beta\}, \{\zeta\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(C-D)}(\Psi) = \{\beta\}$, $U_{\mathcal{R}(C-D)}(\Psi) = \{\alpha, \theta, \delta, \gamma, \beta\}$, $B_{\mathcal{R}(C-D)}(\Psi) = \{\alpha, \delta, \gamma, \theta\}$ then nano topology on $\mathcal{U}$ by C - D, $\tau_{\mathcal{R}(C-D)}(\Psi) = \{\mathcal{U}, \phi, \{\beta\}, \{\alpha, \delta, \beta, \gamma, \theta\}, \{\alpha, \delta, \gamma, \theta\}\}$ and its basis $\mathcal{B}_{(C-D)}(\Psi) = \{\mathcal{U}, \phi, \{\beta\}, \{\alpha, \delta, \gamma, \theta\}\} \neq \mathcal{B}_{(C)}(\Psi)$

ii) Eliminate an attribute **'Medications'** from C.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(C-M)}(\Psi) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta\}, \{\zeta\}, \{\gamma\}, \{\theta\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(C-M)}(\Psi) = \{\delta, \beta, \theta\}$, $U_{\mathcal{R}(C-M)}(\Psi) = \{\delta, \beta, \theta\}$, $B_{\mathcal{R}(C-M)}(\Psi) = \phi$ then nano topology on $\mathcal{U}$ by C - M, $\tau_{\mathcal{R}(C-M)}(\Psi) = \{\mathcal{U}, \phi, \{\delta, \beta, \theta\}\}$ and its basis $\mathcal{B}_{(C-M)}(\Psi) = \{\mathcal{U}, \{\delta, \beta, \theta\}, \phi\} = \mathcal{B}_{(C)}(\Psi)$

iii) Eliminate an attribute **'Water'** from C.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(C-W)}(\Psi) = \{\{\alpha, \gamma\}, \{\eta, \delta\}, \{\beta\}, \{\zeta\}, \{\theta\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(C-W)}(\Psi) = \{\beta, \theta\}$, $U_{\mathcal{R}(C-W)}(\Psi) = \{\eta, \delta, \beta, \theta\}$, $B_{\mathcal{R}(C-W)}(\Psi) = \{\eta, \delta\}$ then nano topology on $\mathcal{U}$ by C - W, $\tau_{\mathcal{R}(C - W)}(\Psi) = \{\mathcal{U}, \phi, \{\beta, \theta\}, \{\eta, \delta, \beta, \theta\}, \{\eta, \delta\}\}$ and its basis $\mathcal{B}_{(C - W)}(\Psi) = \{\mathcal{U}, \phi, \{\beta, \theta\}, \{\eta, \delta\}\} \neq \mathcal{B}_{(C)}(\Psi)$

iv) Eliminate an attribute **'Hair Styling'** from C.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(C - H)}(\Psi) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta, \theta\}, \{\zeta\}, \{\gamma\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(C - H)}(\Psi) = \{\delta, \beta, \theta\}$, $U_{\mathcal{R}(C - H)}(\Psi) = \{\delta, \beta, \theta\}$, $B_{\mathcal{R}(C - H)}(\Psi) = \phi$ then nano topology on $\mathcal{U}$ by C - H, $\tau_{\mathcal{R}(C - H)}(\Psi) = \{\mathcal{U}, \phi, \{\delta, \beta, \theta\}\}$ and its basis $\mathcal{B}_{(C - H)}(\Psi) = \{\mathcal{U}, \{\delta, \beta, \theta\}, \phi\} = \mathcal{B}_{(C)}(\Psi)$

v) Eliminate an attribute **'Stress'** from C.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(C - S)}(\Psi) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta, \lambda\}, \{\zeta\}, \{\gamma\}, \{\theta\}\}$ and the pertaining $L_{\mathcal{R}(C - S)}(\Psi) = \{\delta\}$, $U_{\mathcal{R}(C - S)}(\Psi) = \{\delta, \beta, \lambda, \theta\}$, $B_{\mathcal{R}(C - S)}(\Psi) = \{\beta, \theta, \lambda\}$ then nano topology on $\mathcal{U}$ by C - S, $\tau_{\mathcal{R}(C - S)}(\Psi) = \{\mathcal{U}, \phi, \{\delta\}, \{\delta, \beta, \theta, \lambda\}, \{\beta, \theta, \lambda\}\}$ and its basis $\mathcal{B}_{(C - S)}(\Psi) = \{\mathcal{U}, \phi, \{\delta\}, \{\beta, \theta, \lambda\}\} \neq \mathcal{B}_{(C)}(\Psi)$

STEP 2(A):

Let $F_1 = C - M = \{D, W, H, S\}$ and $\mathcal{U}/\mathcal{R}_{(F1)}(\Psi) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta\}, \{\zeta\}, \{\gamma\}, \{\theta\}, \{\lambda\}\}$ and its basis $\mathcal{B}_{(F1)}(\Psi) = \{\mathcal{U}, \{\delta, \beta, \theta\}, \phi\} = \mathcal{B}_{(C)}(\Psi)$

i) Eliminate an attribute **'Diet'** from F_1.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(F1 - D)}(\Psi) = \{\{\alpha, \theta\}, \{\delta, \gamma\}, \{\eta\}, \{\beta\}, \{\zeta\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(F1 - D)}(\Psi) = \{\beta\}$, $U_{\mathcal{R}(F1 - D)}(\Psi) = \{\alpha, \theta, \delta, \gamma, \beta\}$, $B_{\mathcal{R}(F1 - D)}(\Psi) = \{\alpha, \delta, \gamma, \theta\}$ then nano topology on $\mathcal{U}$ by F_1 - D, $\tau_{\mathcal{R}(F1 - D)}(\Psi) = \{\mathcal{U}, \phi, \{\beta\}, \{\alpha, \delta, \beta, \gamma, \theta\}, \{\alpha, \delta, \gamma, \theta\}\}$ and its basis $\mathcal{B}_{(F1 - D)}(\Psi) = \{\mathcal{U}, \phi, \{\beta\}, \{\alpha, \delta, \gamma, \theta\}\} \neq \mathcal{B}_{(F1)}(\Psi)$

ii) Eliminate an attribute **'Water'** from F_1.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(F1 - W)}(\Psi) = \{\{\alpha, \gamma\}, \{\eta, \delta\}, \{\beta\}, \{\zeta\}, \{\theta\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(F1 - W)}(\Psi) = \{\beta, \theta\}$, $U_{\mathcal{R}(F1 - W)}(\Psi) = \{\eta, \delta, \beta, \theta\}$, $B_{\mathcal{R}(F1 - W)}(\Psi) = \{\eta, \delta\}$ then nano topology on $\mathcal{U}$ by F_1 - W, $\tau_{\mathcal{R}(F1 - W)}(\Psi) = \{\mathcal{U}, \phi, \{\beta, \theta\}, \{\eta, \delta, \beta, \theta\}, \{\eta, \delta\}\}$ and its basis $\mathcal{B}_{(F1 - W)}(\Psi) = \{\mathcal{U}, \phi, \{\beta, \theta\}, \{\eta, \delta\}\} \neq \mathcal{B}_{(F1)}(\Psi)$

iii) Eliminate an attribute **'Hair Styling'** from F_1.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(F1 - H)}(\Psi) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta, \theta\}, \{\zeta\}, \{\gamma\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(F1 - H)}(\Psi) = \{\delta, \beta, \theta\}$, $U_{\mathcal{R}(F1 - H)}(\Psi) = \{\delta, \beta, \theta\}$, $B_{\mathcal{R}(F1 - H)}(\Psi) = \phi$ then nano topology on $\mathcal{U}$ by F_1 - H, $\tau_{\mathcal{R}(F1 - H)}(\Psi) = \{\mathcal{U}, \phi, \{\delta, \beta, \theta\}\}$ and its basis $\mathcal{B}_{(F1 - H)}(\Psi) = \{\mathcal{U}, \{\delta, \beta, \theta\}, \phi\} = \mathcal{B}_{(F1)}(\Psi)$

iv) Eliminate an attribute **'Stress'** from F_1.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(F1 - S)}(\Psi) = \{\{\alpha\}, \{\eta\}, \{\delta, \zeta\}, \{\beta, \lambda\}, \{\gamma\}, \{\theta\}\}$ and the pertaining $L_{\mathcal{R}(F1 - S)}(\Psi) = \{\theta\}$, $U_{\mathcal{R}(F1 - S)}(\Psi) = \{\delta, \beta, \zeta, \theta, \lambda\}$, $B_{\mathcal{R}(F1 - S)}(\Psi) = \{\delta, \beta, \zeta, \lambda\}$ then nano topology on $\mathcal{U}$ by F_1 - S, $\tau_{\mathcal{R}(F1 - S)}(\Psi) = \{\mathcal{U}, \phi, \{\theta\}, \{\delta, \beta, \zeta, \theta, \lambda\}, \{\delta, \beta, \zeta, \lambda\}\}$ and its basis $\mathcal{B}_{(F1 - S)}(\Psi) = \{\mathcal{U}, \phi, \{\theta\}, \{\delta, \beta, \zeta, \lambda\}\} \neq \mathcal{B}_{(F1)}(\Psi)$

STEP 2(B):

Let $F_2 = \mathcal{C} - H = \{D, M, W, S\}$ and $\mathcal{U}/\mathcal{R}_{(F2)}(\Psi) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta, \theta\}, \{\zeta\}, \{\gamma\}, \{\lambda\}\}$ and its basis $\mathcal{B}_{(F2)}(\Psi) = \{\mathcal{U}, \{\delta, \beta, \theta\}, \phi\} = \mathcal{B}_{(C)}(\Psi)$

i) Eliminate an attribute **'Diet'** from F_2.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(F2 - D)}(\Psi) = \{\{\alpha, \beta, \theta\}, \{\eta\}, \{\delta, \gamma\}, \{\zeta\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(F2 - D)}(\Psi) = \phi$, $U_{\mathcal{R}(F2 - D)}(\Psi) = \{\alpha, \beta, \theta, \delta, \gamma\}$, $B_{\mathcal{R}(F2 - D)}(\Psi) = \{\alpha, \beta, \theta, \delta, \gamma\}$ then nano topology on $\mathcal{U}$ by F_2 - D, $\tau_{\mathcal{R}(F2 - D)}(\Psi) = \{\mathcal{U}, \phi, \{\alpha, \beta, \theta, \delta, \gamma\}\}$ and its basis $\mathcal{B}_{(F2 - D)}(\Psi) = \{\mathcal{U}, \phi, \{\alpha, \delta, \beta, \gamma, \theta\}\} \neq \mathcal{B}_{(F2)}(\Psi)$

ii) Eliminate an attribute **'Medications'** from F_2.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(F2 - M)}(\Psi) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta, \theta\}, \{\zeta\}, \{\gamma\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(F2 - M)}(\Psi) = \{\delta, \beta, \theta\}$, $U_{\mathcal{R}(F2 - M)}(\Psi) = \{\delta, \beta, \theta\}$, $B_{\mathcal{R}(C - M)}(\Psi) = \phi$ then nano topology on $\mathcal{U}$ by F_2 - M, $\tau_{\mathcal{R}(F2 - M)}(\Psi) = \{\mathcal{U}, \phi, \{\delta, \beta, \theta\}\}$ and its basis $\mathcal{B}_{(F2 - M)}(\Psi) = \{\mathcal{U}, \phi, \{\delta, \beta, \theta\}\} = \mathcal{B}_{(F2)}(\Psi)$

iii) Eliminate an attribute **'Water'** from F_2.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(F2 - W)}(\Psi) = \{\{\alpha, \gamma\}, \{\eta, \delta\}, \{\beta, \theta\}, \{\zeta\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(F2 - W)}(\Psi) = \{\beta, \theta\}$, $U_{\mathcal{R}(F2 - W)}(\Psi) = \{\eta, \delta, \beta, \theta\}$, $B_{\mathcal{R}(F2 - W)}(\Psi) = \{\eta, \delta\}$ then nano topology on $\mathcal{U}$ by F_2 - W, $\tau_{\mathcal{R}(F2 - W)}(\Psi) = \{\mathcal{U}, \phi, \{\beta, \theta\}, \{\eta, \delta, \beta, \theta\}, \{\eta, \delta\}\}$ and its basis $\mathcal{B}_{(F2 - W)}(\Psi) = \{\mathcal{U}, \phi, \{\beta, \theta\}, \{\eta, \delta\}\} \neq \mathcal{B}_{(F2)}(\Psi)$

iv) Eliminate an attribute **'Stress'** from F_2.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(F2 - S)}(\Psi) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta, \theta, \lambda\}, \{\zeta\}, \{\gamma\}\}$ and the pertaining $L_{\mathcal{R}(F2 - S)}(\Psi) = \{\delta\}$, $U_{\mathcal{R}(F2 - S)}(\Psi) = \{\delta, \beta, \theta, \lambda\}$, $B_{\mathcal{R}(F2 - S)}(\Psi) = \{\beta, \theta, \lambda\}$ then nano topology on $\mathcal{U}$ by F_2 - S, $\tau_{\mathcal{R}(F2 - S)}(\Psi) = \{\mathcal{U}, \phi, \{\delta\}, \{\delta, \beta, \theta, \lambda\}, \{\beta, \theta, \lambda\}\}$ and its basis $\mathcal{B}_{(F2 - S)}(\Psi) = \{\mathcal{U}, \phi, \{\delta\}, \{\beta, \theta, \lambda\}\} \neq \mathcal{B}_{(F2)}(\Psi)$

STEP 3:

Let $P = \mathcal{C} - M, H = \{D, W, S\}$ and $\mathcal{U}/\mathcal{R}_{(P)}(\Psi) = \{\{\alpha\}, \{\eta\}, \{\delta\}, \{\beta, \theta\}, \{\zeta\}, \{\gamma\}, \{\lambda\}\}$ and its basis $\mathcal{B}_{(P)}(\Psi) = \{\mathcal{U}, \{\delta, \beta, \theta\}, \phi\} = \mathcal{B}_{(C)}(\Psi)$

i) Eliminate an attribute **'Diet'** from P.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(P - D)}(\Psi) = \{\{\alpha, \beta, \theta\}, \{\delta, \gamma\}, \{\eta\}, \{\zeta\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(P - D)}(\Psi) = \phi$, $U_{\mathcal{R}(P-D)}(\Psi) = \{\alpha, \beta, \theta, \delta, \gamma\}$, $B_{\mathcal{R}(P - D)}(\Psi) = \{\alpha, \delta, \beta, \gamma, \theta\}$ then nano topology on $\mathcal{U}$ by P - D, $\tau_{\mathcal{R}(P - D)}(\Psi) = \{\mathcal{U}, \phi, \{\alpha, \delta, \beta, \gamma, \theta\}\}$ and its basis $\mathcal{B}_{(P - D)}(\Psi) = \{\mathcal{U}, \phi, \{\alpha, \delta, \beta, \gamma, \theta\}\} \neq \mathcal{B}_{(P)}(\Psi)$

ii) Eliminate an attribute **'Water'** from P.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(P - W)}(\Psi) = \{\{\alpha, \gamma\}, \{\eta, \delta\}, \{\beta, \theta\}, \{\zeta\}, \{\lambda\}\}$ and the pertaining $L_{\mathcal{R}(P - W)}(\Psi) = \{\beta, \theta\}$, $U_{\mathcal{R}(P - W)}(\Psi) = \{\eta, \delta, \beta, \theta\}$, $B_{\mathcal{R}(P - W)}(\Psi) = \{\eta, \delta\}$ then nano topology on $\mathcal{U}$ by P - W, $\tau_{\mathcal{R}(P - W)}(\Psi) = \{\mathcal{U}, \phi, \{\beta, \theta\}, \{\eta, \delta, \beta, \theta\}, \{\eta, \delta\}\}$ and its basis $\mathcal{B}_{(P - W)}(\Psi) = \{\mathcal{U}, \phi, \{\beta, \theta\}, \{\eta, \delta\}\} \neq \mathcal{B}_{(P)}(\Psi)$

iii) Eliminate an attribute **'Stress'** from P.

The pertaining equivalence relation is $\mathcal{U}/\mathcal{R}_{(P - S)}(\Psi) = \{\{\alpha\}, \{\eta\}, \{\delta, \zeta\}, \{\beta, \theta, \lambda\}, \{\gamma\}\}$ and the pertaining $L_{\mathcal{R}(P - S)}(\Psi) = \phi$, $U_{\mathcal{R}(P - S)}(\Psi) = \{\delta, \zeta, \beta, \theta, \lambda\}$, $B_{\mathcal{R}(P - S)}(\Psi) = \{\delta, \beta, \zeta, \theta, \lambda\}$ then nano topology on $\mathcal{U}$ by P - S, $\tau_{\mathcal{R}(P - S)}(\Psi) = \{\mathcal{U}, \phi, \{\delta, \beta, \zeta, \theta, \lambda\}\}$ and its basis $\mathcal{B}_{(P - S)}(\Psi) = \{\mathcal{U}, \phi, \{\delta, \beta, \zeta, \theta, \lambda\}\} \neq \mathcal{B}_{(P)}(\Psi)$.

Thus core = {Diet, Water, Stress}.

Observation

By applying topological reduction of attributes in terms of basis of nano topology we find the main reasons for hair fall among the groups of people are Diet(D), Water(W) and Stress(S) from both the cases. Though every individual is unique they have their own reasons. But diet, water and stress are being the most common core factors for the hair fall problem.

4. Conclusion

This paper defined Nsgb-neighbourhood, Nsgb-adherent points, Nsgb-derived set of Nsgb-sets. The study can be further proceed to discuss about connectedness, compactness, separation axioms on Nsgb-closed (open) sets and can give more applications in decision making problems.

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