COMMON FIXED POINT THEOREMS IN COMPATIBLE MAPPINGS OF TYPE (P*) OF GENERALIZED INTUITIONISTIC FUZZY METRIC SPACES

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ABSTRACT

In this paper, we give some new definition of Compatible mappings of type (P), type (P-1) and type (P-2) in intuitionistic generalized fuzzy metric spaces and prove Common fixed point theorems for six mappings under the conditions of compatible mappings of type (P-1) and type (P-2) in complete intuitionistic fuzzy metric spaces. Our results intuitionistically fuzzify the result of Muthuraj and Pandiselvi [15]

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KEYWORDS

Intuitionistic fuzzy metric spaces, compatible mappings of type (P), type (P-1) and type (P-2), common fixed point.

1. INTRODUCTION


Further, Sedghi and Shobe [19] defined \(\mathcal{M}\)-fuzzy metric space and proved a common fixed point theorem in it. Jong Seo Park [15] introduced the concept of semi compatible and Weak Compatible maps in fuzzy metric space and proved some fixed point theorems satisfying certain conditions in \(\mathcal{M}\)-fuzzy metric spaces.

As a generalization of fuzzy sets, Atanassov [1] introduced and studied the concept of intuitionistic fuzzy sets. Using the idea of intuitionistic fuzzy sets Park [16] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorm as a generalization of fuzzy metric space, George and Veeramani [8] had showed that every metric induces an intuitionistic fuzzy metric and found a necessary and sufficient conditions for an intuitionistic fuzzy metric space to be complete. Choudhary [4] introduced mutually contractive sequence of self maps and proved a fixed point theorem. Kramosil and Michalek [13] introduced the notion of Cauchy sequences in an intuitionistic fuzzy metric space and proved the well known fixed point theorem of Banach[2]. Turkoglu et al [22] gave the generalization of Jungck’s[11] Common fixed point theorem to intuitionistic fuzzy metric spaces.
In this paper, we extend the result of common fixed point theorem for compatible mappings of type (P-1) and type (P-2) in intuitionistic fuzzy metric space and prove common fixed point theorem of type (P-1) and type (P-2) in intuitionistic fuzzy metric spaces, we also give an example to validate our main theorem. Our results intuitionistically fuzzify the result of Muthuraj and Pandiselvi [15].

2. PRELIMINARIES

We start with the following definitions.

Definition 2.1

A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is said to be a continuous t-norm if $*$ satisfies the following conditions.

(i) $*$ is commutative and associative,

(ii) $*$ is continuous,

(iii) $a * 1 = a$ for all $a \in [0,1],$

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a,b,c,d \in [0,1].$

Definition 2.2

A binary operation $\cdot : [0,1] \times [0,1] \to [0,1]$ is said to be a continuous t-conorm if $\cdot$ satisfies the following conditions:

(i) $\cdot$ is commutative and associative,

(ii) $\cdot$ is continuous,

(iii) $a \cdot 0 = a$ for all $a \in [0,1],$

(iv) $a \cdot b \leq c \cdot d$ whenever $a \leq c$ and $b \leq d$ for all $a,b,c,d \in [0,1].$

Definition 2.3

A 5-tuple $(X, \mathcal{M}, \mathcal{N}, *, \cdot)$ is called an intuitionistic fuzzy metric space if $X$ is an arbitrary set, $*$ is a continuous t-norm, $\cdot$ a continuous t-conorm and $\mathcal{M}, \mathcal{N}$ are fuzzy sets on $X^3 \times (0, \infty)$, satisfying the following conditions, for each $x, y, z, a \in X$ and $t, s > 0$.

a) $\mathcal{M}(x, y, z, t) + \mathcal{N}(x, y, z, t) \leq 1.$

b) $\mathcal{M}(x, y, z, t) > 0.$

c) $\mathcal{M}(x, y, z, t) = 1$ if and only if $x = y = z.$

d) $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$ where $p$ is a permutation function,

e) $\mathcal{M}(x, y, a, t) * \mathcal{M}(a, z, z, s) \leq \mathcal{M}(x, y, z, t + s)$

f) $\mathcal{M}(x, y, z) : (0, \infty) \to [0, 1]$ is continuous

g) $\mathcal{N}(x, y, z, t) > 0$

h) $\mathcal{N}(x, y, z, t) = 0$, if and only if $x = y = z,$

i) $\mathcal{N}(x, y, z, t) = \mathcal{N}(p\{x, y, z\}, t)$ where $p$ is a permutation function,

j) $\mathcal{N}(x, y, a, t) \cdot \mathcal{N}(a, z, z, s) \geq \mathcal{N}(x, y, z, t + s)$,

k) $\mathcal{N}(x, y, z, \cdot) : (0, \infty) \to [0, 1]$ is continuous.

Then $(\mathcal{M}, \mathcal{N})$ is called an intuitionistic fuzzy metric on $X.$
Example 2.4

Let \( X = \mathbb{R} \), and 
\[
\mathcal{M}(x, y, z, t) = \frac{t}{|x+y|+|y-z|+|z-x|},
\]
\[
\mathcal{N}(x, y, z, t) = \frac{|x-y|+|y-z|+|z-x|}{t+|x-y|+|y-z|+|z-x|},
\]
for every \( x, y, z > 0 \), let \( A \) and \( B \) defined as \( Ax = 2x + 1 \), \( Bx = x + 2 \), consider the sequence \( x_n = \frac{1}{n} + 1, n = 1, 2, \ldots \) Thus we have

\[
\lim_{n \to \infty} \mathcal{M}(Ax_n, 3, 3, t) = \lim_{n \to \infty} \mathcal{M}(Bx_n, 3, 3, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} \mathcal{N}(Ax_n, 3, 3, t) = \lim_{n \to \infty} \mathcal{N}(Bx_n, 3, 3, t) = 0, \text{ for every } t > 0.
\]

Then \( A \) and \( B \) satisfying the property (E).

Definition 2.5

Let \((X, \mathcal{M}, \mathcal{N}, *, \otimes)\) be an intuitionistic fuzzy metric space and \( \{x_n\} \) be a sequence in \( X \).

a) \( \{x_n\} \) is said to be converges to a point \( x \in X \), if
\[
\lim_{n \to \infty} \mathcal{M}(x, x, x_n, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} \mathcal{N}(x, x, x_n, t) = 0, \text{ for all } t > 0.
\]
b) \( \{x_n\} \) is called Cauchy sequence if
\[
\lim_{n \to \infty} \mathcal{M}(x_{n+p}, x_{n+p}, x_n, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} \mathcal{N}(x_{n+p}, x_{n+p}, x_n, t) = 0 \quad \text{for all } t > 0 \text{ and } p > 0.
\]
c) An intuitionistic fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Lemma 2.6

Let \((X, \mathcal{M}, \mathcal{N}, *, \otimes)\) be an intuitionistic fuzzy metric space. Then \( \mathcal{M}(x, y, z, t) \) and \( \mathcal{N}(x, y, z, t) \) are non-decreasing with respect to \( t \), for all \( x, y, z \) in \( X \).

Proof

By definition 2.3, for each \( x, y, z, a \in X \) and \( t, s > 0 \)
we have \( \mathcal{M}(x, y, a, t) \ast \mathcal{M}(a, z, z, s) \leq \mathcal{M}(x, y, z, t + s) \). If we set \( a = z \),
we get \( \mathcal{M}(z, y, z, t) \ast \mathcal{M}(z, z, z, s) \leq \mathcal{M}(x, y, z, t + s) \), that is
\[
\mathcal{M}(x, y, z, t + s) \geq \mathcal{M}(x, y, z, t).
\]
Similarly, \( \mathcal{N}(x, y, a, t) \otimes \mathcal{N}(a, z, z, s) \geq \mathcal{N}(x, y, z, t + s) \), for each \( x, y, z, a \in X \) and \( t, s > 0 \), by definition of \((X, \mathcal{N}, \otimes)\). If we set \( a = z \), we get
\[
\mathcal{N}(x, y, z, t) \otimes \mathcal{N}(z, z, z, s) \geq \mathcal{N}(x, y, z, t + s)
\]
that is \( \mathcal{N}(x, y, z, t + s) \leq \mathcal{N}(x, y, z, t) \). Hence in IFMS \((X, \mathcal{M}, \mathcal{N}, *, \otimes)\),
\( \mathcal{M}(x, y, z, t) \) and \( \mathcal{N}(x, y, z, t) \) are non-decreasing with respect to \( t \), for all \( x, y, z \) in \( X \).
3. COMPATIBLE MAPPINGS OF TYPE

Definition 3.1

Let A and S be self mappings from an intuitionistic fuzzy metric space \((X, \mathcal{M}, \mathcal{N}, *, \odot)\) into itself. Then the mappings are said to be compatible if

\[
\lim_{n \to \infty} \mathcal{M}(A S x_n, S A x_n, S A x_n, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} \mathcal{N}(A S x_n, S A x_n, S A x_n, t) = 0,
\]

for all \(t > 0\) whenever \(\{x_n\}\) is a sequence in \(X\) such that \(\lim_{n \to \infty} A x_n = \lim_{n \to \infty} S x_n = z\) for some \(z \in X\).

Definition 3.2

Let A and S be self mappings from an intuitionistic fuzzy metric space \((X, \mathcal{M}, \mathcal{N}, *, \odot)\) into itself. Then the mappings are said to be compatible of type (P), if

\[
\lim_{n \to \infty} \mathcal{M}(A A x_n, S S x_n, S S x_n, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} \mathcal{N}(A A x_n, S S x_n, S S x_n, t) = 0
\]

for all \(t > 0\), whenever \(\{x_n\}\) is a sequence in \(X\) such that \(\lim_{n \to \infty} A x_n = \lim_{n \to \infty} S x_n = z\) for some \(z \in X\).

Definition 3.3

Let A and S be self mappings from an intuitionistic fuzzy metric space \((X, \mathcal{M}, \mathcal{N}, *, \odot)\) into itself. Then the mappings are said to be R-Weakly commuting of type (P), if there exists some \(R > 0\), such that

\[
\mathcal{M}(A A x, S S x, S S x, t) \geq \mathcal{M}(A x, S x, S x, \frac{t}{R}),
\]

\[
\mathcal{N}(A A x, S S x, S S x, t) \leq \mathcal{N}(A x, S x, S x, \frac{t}{R}),
\]

for all \(x \in X\) and \(t > 0\).

Definition 3.4

Let A and S be self mappings from an intuitionistic fuzzy metric space \((X, \mathcal{M}, \mathcal{N}, *, \odot)\) into itself. Then the mappings are said to be compatible of type (P-1) if

\[
\lim_{n \to \infty} \mathcal{M}(S A x_n, A A x_n, A A x_n, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} \mathcal{N}(S A x_n, A A x_n, A A x_n, t) = 0
\]

for all \(t > 0\), whenever \(\{x_n\}\) is a sequence in \(X\) such that \(\lim_{n \to \infty} A x_n = \lim_{n \to \infty} S x_n = z\) for some \(z \in X\).

Definition 3.5

Let A and S be self mappings from an intuitionistic fuzzy metric space \((X, \mathcal{M}, \mathcal{N}, *, \odot)\) into itself. Then the mappings are said to be compatible of type (P-2) if

\[
\lim_{n \to \infty} \mathcal{M}(A A x_n, S S x_n, S S x_n, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} \mathcal{N}(A A x_n, S S x_n, S S x_n, t) = 0
\]

for all \(t > 0\) whenever \(\{x_n\}\) is a sequence in \(X\) such that \(\lim_{n \to \infty} A x_n = \lim_{n \to \infty} S x_n = z\) for some \(z \in X\).
Proposition 3.6

Let $A$ and $S$ be self mappings from an intuitionistic fuzzy metric space $(X, \mathcal{M}, \mathcal{N}, *, \diamond)$ into itself.

a) If $A$ is a continuous map then the pair of mappings $(A, S)$ is compatible of type (P-1) if and only if $A$ and $S$ are compatible.

b) If $S$ is a continuous map then the pair of mappings $(A, S)$ is compatible of type (P-2) if and only if $A$ and $S$ are compatible.

Proof

a) Let $\lim_{n \to \infty} Ax_n = \lim_{n \to \infty} Sx_n = z$ for some $z \in X$, and let the pair $(A, S)$ be compatible of type (P-1). Since $A$ is continuous, we have $\lim_{n \to \infty} ASx_n = Az$ and $\lim_{n \to \infty} AAx_n = Az$. Therefore it follows that

\[
\mathcal{M}(SAx_n, ASx_n, ASx_n, t) \geq 1 \ast 1 = 1 \text{ and } \lim_{n \to \infty} \mathcal{M}(SAx_n, ASx_n, ASx_n, t) = 0 \diamond 0 = 0
\]

and so the mappings $A$ and $S$ are compatible.

Now, let $A$ and $S$ be compatible. Therefore it follows that

\[
\mathcal{M}(SAx_n, AAx_n, AAx_n, t) \geq 1 \ast 1 = 1 \text{ and } \lim_{n \to \infty} \mathcal{M}(SAx_n, AAx_n, AAx_n, t) = 0 \diamond 0 = 0
\]

so that pair of mappings $(A, S)$ are compatible of type (P-1).
b) Let \( \lim_{n \to \infty} Sx_n = \lim_{n \to \infty} Ax_n = z \) for some \( z \) in \( X \) and let the pair \((A, S)\) be compatible of type (P-2). Since \( S \) is continuous, we have \( \lim_{n \to \infty} SAx_n = Sz \) and \( \lim_{n \to \infty} SSx_n = Sz \). Therefore it follows that
\[
\mathcal{M}(SAx_n, ASx_n, ASx_n, t) \geq \mathcal{M}(SSx_n, ASx_n, ASx_n, t/2)
\]
\[
\ast \mathcal{M}(SSx_n, ASx_n, ASx_n, t/2)
\]
and
\[
\mathcal{N}(SAx_n, ASx_n, ASx_n, t) \leq \mathcal{N}(SSx_n, SSx_n, SSx_n, t/2)
\]
\[
\diamond \mathcal{N}(SSx_n, SSx_n, SSx_n, t/2)
\]
yields \( \lim_{n \to \infty} \mathcal{M}(SAx_n, ASx_n, ASx_n, t) \geq 1 \ast 1 = 1 \) and
\( \lim_{n \to \infty} \mathcal{N}(SAx_n, ASx_n, ASx_n, t) \leq 0 \diamond 0 = 0 \) and so the mappings \( A \) and \( S \) are compatible.

Now let \( A \) and \( S \) be compatible. Then we have
\[
\mathcal{M}(ASx_n, SSx_n, SSx_n, t) \geq \mathcal{M}(ASx_n, SAx_n, SAx_n, t/2)
\]
\[
\ast \mathcal{M}(SAx_n, SSx_n, SSx_n, t/2)
\]
and
\[
\mathcal{N}(ASx_n, SSx_n, SSx_n, t) \leq \mathcal{N}(ASx_n, SAx_n, SAx_n, t/2)
\]
\[
\diamond \mathcal{N}(ASx_n, SAx_n, SAx_n, t/2)
\]
yields \( \lim_{n \to \infty} \mathcal{M}(ASx_n, SSx_n, SSx_n, t) \geq 1 \ast 1 = 1 \) and
\( \lim_{n \to \infty} \mathcal{N}(ASx_n, SSx_n, SSx_n, t) \leq 0 \diamond 0 = 0 \) and so the pair of mappings \((A, S)\) are compatible of type (P-2).

**Proposition 3.7**

Let \( A \) and \( S \) be self mappings from an intuitionistic fuzzy metric space \((X, \mathcal{M}, \mathcal{N}, \ast, \diamond)\) into itself. If the pair \((A, S)\) is compatible of type (P-2) and \( Sz = Az \) for some \( z \in X \). Then \( ASz = SSz \).

**Proof:**

Let \( \{ x_n \} \) be a sequence in \( X \) defined by \( x_n = z \) for \( n = 1, 2, \ldots \) and let \( Az = Sz \).

Then we have \( \lim_{n \to \infty} Sx_n = Sz \) and \( \lim_{n \to \infty} Ax_n = Az \). Since the pair \((A, S)\) is compatible of type (P-2), we have
\[
\mathcal{M}(ASz, SSz, SSz, t) = \lim_{n \to \infty} \mathcal{M}(ASx_n, SSx_n, SSx_n, t) = 1 \text{ and}
\]
\[
\mathcal{N}(ASz, SSz, SSz, t) = \lim_{n \to \infty} \mathcal{N}(ASx_n, SSx_n, SSx_n, t) = 0.
\]
Hence \( ASz = SSz \).
Proposition 3.8

Let $A$ and $S$ self mappings from an intuitionistic fuzzy metric space $(X, \mathcal{M}, \mathcal{N}, *, \diamond)$ with $t * t \geq t$ and $(1-t) \diamond (1-t) \leq 1-t$ for all $t \in [0, 1]$ if the pair $(A, S)$ are compatible of type $(p-1)$ and $Ax_{n}, Sx_{n} \to z$ for some $z$ in $X$ and a sequence $\{x_{n}\}$ in $X$.

Then $AAx_{n} \to Sz$, if $S$ is continuous at $z$.

Proof

Since $S$ is continuous at $z$, we have $SAx_{n} \to Sz$. Since the pair $(A, S)$ are compatible of type (P-1), we have $\mathcal{M}(SAx_{n}, AAx_{n}, AAx_{n}, t) \to 1$ as $n \to \infty$. It follows that

\[
\mathcal{M}(Sz, AAx_{n}, AAx_{n}, t) \geq \mathcal{M}(Sz, SAx_{n}, SAx_{n}, l) \ast \mathcal{M}(SAx_{n}, AAx_{n}, AAx_{n}, l) \quad \text{and}
\]

\[
\mathcal{N}(Sz, AAx_{n}, AAx_{n}, t) \leq \mathcal{N}(Sz, SAx_{n}, SAx_{n}, l) \ast \mathcal{N}(SAx_{n}, AAx_{n}, AAx_{n}, l)
\]

yield

\[
\mathcal{M}(Sz, AAx_{n}, AAx_{n}, t) \geq 1 \ast 1 = 1 \quad \text{and}
\]

\[
\mathcal{N}(Sz, AAx_{n}, AAx_{n}, t) \leq 0 \diamond 0 = 0
\]

so we have $AAx_{n} \to Sz$ as $n \to \infty$.

Proposition 3.9

Let $A$ and $S$ be self mappings from an intuitionistic fuzzy metric space $(X, \mathcal{M}, \mathcal{N}, *, \diamond)$ with $t * t \geq t$ and $(1-t) \diamond (1-t) \leq 1-t$ for $t \in [0, 1]$. If the pair $(A, S)$ are compatible of type $(P-2)$ and $Ax_{n}, Sx_{n} \to z$ for some $z$ in $X$ and sequence $\{x_{n}\}$ in $X$. Then $SSx_{n} \to Az$ if $A$ is continuous at $z$.

Proof

Since $A$ is continuous at $z$, we have $ASx_{n} \to Az$. Since the pair $(A, S)$ are compatible of type (P-2), we have $\mathcal{M}(ASx_{n}, SSx_{n}, SSx_{n}, t) \to 1$ as $n \to \infty$, it follows that

\[
\mathcal{M}(Az, SSx_{n}, SSx_{n}, t) \geq \mathcal{M}(Az, ASx_{n}, ASx_{n}, l) \ast \mathcal{M}(ASx_{n}, SSx_{n}, SSx_{n}, l) \quad \text{and}
\]

\[
\mathcal{N}(Az, SSx_{n}, SSx_{n}, t) \leq \mathcal{N}(Az, ASx_{n}, ASx_{n}, l) \diamond \mathcal{N}(ASx_{n}, SSx_{n}, SSx_{n}, l)
\]

yield

\[
\lim_{n \to \infty} \mathcal{M}(Az, SSx_{n}, SSx_{n}, t) \geq 1 \ast 1 = 1 \quad \text{and}
\]

\[
\lim_{n \to \infty} \mathcal{N}(Az, SSx_{n}, SSx_{n}, t) \leq 0 \diamond 0 = 0
\]

so we have $SSx_{n} \to Az$ as $n \to \infty$. 
4. MAIN RESULTS

Theorem 4.1

Let \((X, \mathcal{M}, \mathcal{N}, *, \diamond)\) be a complete generalized intuitionistic fuzzy metric space and let \(A, B, P, Q, S\) and \(T\) be self mappings of \(X\) satisfying the following conditions.

- (i) \(P(X) \subseteq ST(X), Q(X) \subseteq AB(X)\)
- (ii) The pair \((P, AB)\) and \((Q, ST)\) are compatible mappings of type (P)
- (iii) \(ST\) is continuous
- (iv) \(\mathcal{M}(Px, Qz, Qz, qt) \geq \min \{\mathcal{M}(ABx, Py, Qy, t), \mathcal{M}(ABx, Py, STz, t), \mathcal{M}(Qy, STz, Py, t), \mathcal{M}(ABx, Qy, STz, t)\}\)

Then the mappings \(P, Q, AB\) and \(ST\) have a unique common fixed point in \(X\).

Proof

Let \(x_0\) be any arbitrary point in \(X\). Thus we construct a sequence \(\{y_n\}\) in \(X\) such that \(y_{2n-1} = STx_{2n-1} = Px_{2n-2}\) and \(y_{2n} = ABx_{2n} = Qx_{2n-1}\). Put \(x = x_{2n-1}, y = x_{2n-1}, z = x_{2n}\).

\[
\begin{align*}
\mathcal{M}(Px_{2n-1}, Qx_{2n}, Qx_{2n}, qt) & \geq \min \left\{ \mathcal{M}(ABx_{2n-1}, Px_{2n-1}, Qx_{2n-1}, t), \mathcal{M}(ABx_{2n-1}, Px_{2n-1}, STx_{2n}, t), \mathcal{M}(Qx_{2n-1}, STx_{2n}, Px_{2n-1}, t), \mathcal{M}(ABx_{2n-1}, Qx_{2n-1}, STx_{2n}, t) \right\} \\
\mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, qt) & \geq \min \left\{ \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t), \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t), \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) \right\}
\end{align*}
\]

This implies that \(\mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t)\) is an increasing sequence of positive real numbers.

\[
\begin{align*}
\mathcal{N}(Px_{2n-1}, Qx_{2n}, Qx_{2n}, qt) & \leq \max \left\{ \mathcal{N}(ABx_{2n-1}, Px_{2n-1}, Qx_{2n-1}, t), \mathcal{N}(ABx_{2n-1}, Px_{2n-1}, STx_{2n}, t), \mathcal{N}(Qx_{2n-1}, STx_{2n}, Px_{2n-1}, t), \mathcal{N}(ABx_{2n-1}, Qx_{2n-1}, STx_{2n}, t) \right\} \\
\mathcal{N}(y_{2n}, y_{2n+1}, y_{2n+1}, qt) & \leq \max \left\{ \mathcal{N}(y_{2n-1}, y_{2n}, y_{2n}, t), \mathcal{N}(y_{2n-1}, y_{2n}, y_{2n}, t), \mathcal{N}(y_{2n-1}, y_{2n}, y_{2n}, t) \right\}
\end{align*}
\]

This implies that \(\mathcal{N}(y_{2n}, y_{2n+1}, y_{2n+1}, t)\) is a decreasing sequence of positive real numbers.

Now to prove that \(\mathcal{M}(y_{n}, y_{n+1}, y_{n+1}, t)\) converges to 1 as \(n \to \infty\) and \(\mathcal{N}(y_{2n}, y_{2n+1}, y_{2n+1}, t)\) converges to 0 as \(n \to \infty\). By lemma 2.6,
Thus by the definition of IFMS,

\[ M(\{y_n\}, q) \geq M(\{y_{n-1}\}, q) \geq M(\{y_{n-2}\}, q) \]

\[
\vdots \geq M(\{y_0\}, q)
\]

Thus \[ M(\{y_n\}, q) \geq M(\{y_{n+1}\}, q) \]

Then by the definition of IFMS,

\[ M(\{y_n\}, y_{n+1}, t) \leq M(\{y_{n-1}\}, y_n, t) \leq M(\{y_{n-2}\}, y_{n-1}, t) \]

\[
\vdots \leq M(\{y_0\}, y_1, t)
\]

Thus \[ M(\{y_n\}, y_{n+1}, t) \leq M(\{y_{n-1}\}, y_n, t) \leq M(\{y_{n-2}\}, y_{n-1}, t) \]

\[
\vdots \leq M(\{y_0\}, y_1, t)
\]

Then by the definition of IFMS,

\[ N(\{y_n\}, y_{n+1}, t) \leq N(\{y_{n-1}\}, y_n, t) \leq N(\{y_{n-2}\}, y_{n-1}, t) \]

\[
\vdots \leq N(\{y_0\}, y_1, t)
\]

Then by the definition of IFMS,

\[ N(\{y_n\}, y_{n+1}, t) \leq N(\{y_{n-1}\}, y_n, t) \leq N(\{y_{n-2}\}, y_{n-1}, t) \]

\[
\vdots \leq N(\{y_0\}, y_1, t)
\]

Thus \( \{y_n\} \) is a Cauchy sequence in intuitionistic fuzzy metric space \( X \).

Since \( X \) is complete, there exists a point \( u \in X \) such that \( y_n \to u \).

Thus \( \{ABx_{2n}\}, \{Qx_{2n-1}\}, \{STx_{2n-1}\}, \{Px_{2n-2}\} \) are Cauchy sequence converge to \( u \).

Put \( x = ABx_{2n}, y = u, z = STx_{2n-1} \) in (iv), we get

\[ M(ABx_{2n}, Pu, StSTx_{2n-1}, t) \geq M(ABx_{2n}, Pu, StSTx_{2n-1}, t) \]

\[ N(ABx_{2n}, Pu, StSTx_{2n-1}, t) \leq N(ABx_{2n}, Pu, StSTx_{2n-1}, t) \]
Now take the limit as \( n \to \infty \) and using (ii), we get,

\[
\mathcal{M}(P_u, Q_u, Q_u, q_t) \geq \min \{ \mathcal{M}(P_u, P_u, Q_u, t), \mathcal{M}(Q_u, P_u, Q_u, t), \mathcal{M}(P_u, Q_u, Q_u, t) \}
\]

and

\[
\mathcal{N}(P_u, Q_u, Q_u, q_t) \leq \max \{ \mathcal{N}(P_u, P_u, Q_u, t), \mathcal{N}(Q_u, Q_u, P_u, t), \mathcal{N}(Q_u, P_u, Q_u, t) \}.
\]

Then by lemma 2.6, we get

\[
\mathcal{M}(P_u, Q_u, Q_u, q_t) \geq \mathcal{M}(Q_u, P_u, Q_u, t) \quad \text{and} \quad \mathcal{N}(P_u, Q_u, Q_u, q_t) \leq \mathcal{N}(P_u, Q_u, Q_u, t).
\]

Therefore \( P_u = Q_u \). Now put \( x = ABx_{2n}, \ y = x_{2n-1}, \ z = x_{2n-1}, \) in (iv), we get

\[
\mathcal{M}(PABx_{2n}, Qx_{2n-1}, Qx_{2n-1}, q_t) \geq \min \{ \mathcal{M}(ABABx_{2n}, P_{x_2n-1}, Q_{x_2n-1}, t), \mathcal{M}(ABABx_{2n}, Q_{x_2n-1}, P_{x_2n-1}, ST_{x_2n-1}, t), \mathcal{M}(Qx_{2n-1}, ST_{x_2n-1}, P_{x_2n-1}, t), \mathcal{M}(Qx_{2n-1}, ST_{x_2n-1}, Q_{x_2n-1}, t) \}
\]

and

\[
\mathcal{N}(PABx_{2n}, Qx_{2n-1}, Qx_{2n-1}, q_t) \leq \max \{ \mathcal{N}(ABABx_{2n}, P_{x_2n-1}, Q_{x_2n-1}, t), \mathcal{N}(Qx_{2n-1}, ST_{x_2n-1}, P_{x_2n-1}, t), \mathcal{N}(Qx_{2n-1}, ST_{x_2n-1}, Q_{x_2n-1}, t) \}.
\]

Thus we have \( \mathcal{M}(P_u, u, u, q_t) \geq \mathcal{M}(P_u, u, u, t) \) and

\[
\mathcal{N}(P_u, u, u, q_t) \geq \mathcal{N}(P_u, u, u, t).
\]

Therefore \( P_u = u \). This implies \( P_u = Q_u = u \).

Now put \( x = P_{x_2n-2}, \ y = P_{x_2n-2}, \ z = u \) in (iv), we get

\[
\mathcal{M}(PPx_{2n-2}, Q_u, Q_u, q_t) \geq \min \{ \mathcal{M}(ABPx_{2n-2}, PPx_{2n-2}, Q_{P_{x_2n-2}}, t), \mathcal{M}(ABPx_{2n-2}, PPx_{2n-2}, ST_u, t), \mathcal{M}(Q_{P_{x_2n-2}}, PPx_{2n-2}, ST_u, t), \mathcal{M}(ABPx_{2n-2}, Q_{P_{x_2n-2}}, ST_u, t) \}
\]
Now taking the limit as \( n \to \infty \) and on using (ii) and (iii), we get
\[
\mathcal{N}(ABu, u, u, qt) \geq \min\left\{ \mathcal{M}(Qu, u, ABu, t), \mathcal{M}(ABu, Qu, u, t) \right\}
\]
This implies
\[
\mathcal{M}(ABu, u, u, qt) \geq \mathcal{M}(u, u, STu, t)
\]
Therefore by lemma (2.6) we have \( ABu = u \). Thus \( Pu = Qu = ABu = u \).

On using lemma (2.6) we have
\[
\mathcal{M}(STu, u, STu, t) \geq \mathcal{M}(STu, STu, u, t) \quad \text{and} \quad \mathcal{N}(STu, u, STu, t) \leq \mathcal{N}(STu, STu, u, t).
\]
Thus \( STu = u \). We get \( Pu = Qu = ABu = STu = u \).

**Uniqueness**

Let \( w \) be another common fixed point of \( A, B, P, Q, S \) and \( T \). Then
\[
\mathcal{M}(Pu, Qw, Qw, qt) \geq \min\left\{ \mathcal{M}(ABu, Pw, Qw, t), \mathcal{M}(ABu, STw, t) \right\}
\]
\[ M(u, w, w, qt) \geq \min \{ M(u, w, w, t), M(u, w, w, t) \} \]
\[ M(u, w, w, qt) \geq M(u, w, w, t) \]
\[ N(Pu, Qw, Qw, qt) \leq \max \{ N(ABu, Pw, Qw, t), N(ABu, Pw, STw, t), N(ABu, Qw, STw, t) \} \]
\[ N(u, w, w, qt) \leq \max \{ N(u, w, w, t), N(u, w, w, t), N(u, w, w, t), N(u, w, w, t) \} \]

which is a contradiction. Therefore \( u = w \).

Hence the common fixed point is unique.

**Corollary 4.2**

Let \((X, M, \mathcal{N}, *, \circ)\) be a complete generalized intuitionistic fuzzy metric space and let \(A, P, Q\) and \(S\) be self mappings of \(X\) satisfying the following conditions.

(i) \(P(X) \subseteq S(X), Q(X) \subseteq A(X)\)

(ii) The pair \((P, A)\) and \((Q, S)\) are compatible mappings of type \((P)\)

(iii) \(S\) is continuous

(iv) \[ M(Px, Qz, Qz, qt) \geq \min \{ M(Ax, Py, Qy, t), M(Ax, Py, Sz, t), M(Qy, Sz, Py, t), M(Ax, Qy, Sz, t) \} \]

\[ N(Px, Qz, Qz, qt) \leq \max \{ N(Ax, Py, Qy, t), N(Ax, Py, Sz, t), N(Qy, Sz, Py, t), N(Ax, Qy, Sz, t) \} \]

Then the mappings \(P, Q, A\) and \(S\) have a unique common fixed point in \(X\).

**Corollary 4.3**

Let \((X, M, \mathcal{N}, *, \circ)\) be a complete generalized intuitionistic fuzzy metric space and let \(B, P, Q, T\) be self mappings of \(X\) satisfying the conditions (i), (ii), (iii), & (iv) with \(S = I\) and \(A = I\);

Then the mappings \(B, P, Q\) and \(T\) have a unique common fixed point.

**Corollary 4.4**

Let \((X, M, \mathcal{N}, *, \circ)\) be a complete generalized intuitionistic fuzzy metric space and let \(A, B, P, Q, S, T\) be self mappings of \(X\) satisfying the following conditions:

(i) \(P(X) \subseteq ST(X), Q(X) \subseteq AB(X)\)

(ii) The pair \((P, AB)\) and \((Q, ST)\) are compatible mappings of type \((P)\)

(iii) \(ST\) is continuous

(iv) \[ M(Px, Qz, Qz, qt) \geq M(ABx, Py, Qy, t) * M(ABx, Py, STz, t) * M(Qy, STz, Py, t) * M(ABx, Qy, STz, t) \]

\[ N(Px, Qz, Qz, qt) \leq N(ABx, Py, Qy, t) * N(ABx, Py, STz, t) \]

Then the mappings \(P, Q, AB\) and \(ST\) have a unique common fixed point in \(X\).
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