MODIFICATION OF DOPANT CONCENTRATION PROFILE IN A FIELD-EFFECT HETERO-TRANSISTOR FOR MODIFICATION ENERGY BAND DIAGRAM

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ABSTRACT

In this paper we consider an approach of manufacturing more compact field-effect heterotransistors. The approach based on manufacturing a heterostructure, which consist of a substrate and an epitaxial layer with specific configuration. After that several areas of the epitaxial layer have been doped by diffusion or ion implantation with optimized annealing of dopant and/or radiation defects. At the same time we introduce an approach of modification of energy band diagram by additional doping of channel of the transistors. We also consider an analytical approach to model and optimize technological process.

KEYWORDS

Modification of profile of dopant; decreasing of dimension of field-effect transistor; modification of energy band diagram

1. INTRODUCTION

Development of solid state electronic leads to increasing performance of the appropriate electronic devices [1-11]. At the same time one can find increasing integration rate of integrated circuits [1-3,5,7]. In this situation dimensions of elements of integrated circuits decreases. To increase performance of solid-state electronics devices are now elaborating new technological processes of manufacturing of solid state electronic devices. Another ways to increase the performance are optimization of existing technological processes and determination new materials with higher values of charge carriers mobilities. To decrease dimensions of elements of integrated circuits they are elaborating new and optimizing existing technological processes. Framework this paper we introduce an approach to decrease dimensions of field-effect heterotransistors. At the same time we introduce an approach of modification of energy band diagram for regulation of transport of charge carriers. The approach based on manufacturing a field-effect transistor in the heterostructure from Fig. 1. The heterostructure consists of a substrate and an epitaxial layer. They are have been considered four sections in the epitaxial layer. The sections have been doped by diffusion or ion implantation. Left and right sections will be considered in future as source and drain, respectively. Both average sections became as channel of transistor. Using one section instead two sections leads to simplification of structure of transistor. However using additional doped section framework the channel of the considered transistor gives us possibility to modify energy band diagram in the structure. After finishing of the considered doping annealing of dopant and/or radiation defects should be done. Several conditions for achievement of decreasing of dimensions of the field-effect transistor have been formulated.

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2. METHOD OF SOLUTION

To solve our aim we determine spatio-temporal distributions of concentrations of dopants. We calculate the required distributions by solving the second Fick’s law in the following form [12,13]

\[
\frac{\partial C(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_c \frac{\partial C(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_c \frac{\partial C(x,y,z,t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ D_c \frac{\partial C(x,y,z,t)}{\partial z} \right].
\]

(1)

Boundary and initial conditions for the equations are

\[
\begin{align*}
\frac{\partial C(x,y,z,t)}{\partial x} \bigg|_{x=0} &= 0, & \frac{\partial C(x,y,z,t)}{\partial x} \bigg|_{x=L_x} &= 0, & \frac{\partial C(x,y,z,t)}{\partial y} \bigg|_{y=0} &= 0, & \frac{\partial C(x,y,z,t)}{\partial y} \bigg|_{y=L_y} &= 0, & \frac{\partial C(x,y,z,t)}{\partial z} \bigg|_{z=0} &= 0, & \frac{\partial C(x,y,z,t)}{\partial z} \bigg|_{z=L_z} &= 0, & C(x,y,z,0) &= f(x,y,z). \\
\end{align*}
\]

(2)

Here the function \( C(x,y,z,t) \) describes the distribution of concentration of dopant in space and time. \( D_c \) describes distribution the dopant diffusion coefficient in space and as a function of temperature of annealing. Dopant diffusion coefficient will be changed with changing of materials of heterostructure, heating and cooling of heterostructure during annealing of dopant or radiation defects (with account Arrhenius law). Dependences of dopant diffusion coefficient on coordinate in heterostructure, temperature of annealing and concentrations of dopant and radiation defects could be written as [14-16]

\[
D_c = D_t \left( x, y, z, T \right) \left[ 1 + \gamma_1 \frac{C^* (x,y,z,t)}{P^* (x,y,z,T)} \right] \left[ 1 + \gamma_2 \frac{V(x,y,z,t)}{V^*} \right].
\]

(3)

Here function \( D_t (x,y,z,T) \) describes dependences of dopant diffusion coefficient on coordinate and temperature of annealing \( T \). Function \( P (x,y,z,T) \) describes the same dependences of the limit of solubility of dopant. The parameter \( \gamma \) is integer and usually could be varying in the following interval \( \gamma \in [1,3] \). The parameter describes quantity of charged defects, which interacting (in aver-
Here we consider temperature dependence of point radiation defects and vacancy generation of atoms. As long as temperature is high enough, the equilibrium distribution of vacancies can be calculated as 

\[ \rho(x,y,z,t) = f_p(x,y,z). \]  

(5)

It is known, that doping of materials by diffusion did not leads to radiation damage of materials. In this situation \( \zeta_1 = \zeta_2 = 0 \). We determine spatio-temporal distributions of concentrations of radiation defects by solving the following system of equations [15,16]

\[
\frac{\partial I(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_x(x,y,z,T) \frac{\partial I(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_y(x,y,z,T) \frac{\partial I(x,y,z,t)}{\partial y} \right] - k_{l,v}(x,y,z,T) I(x,y,z,t) \times \]

\[
\times I^2(x,y,z,t) + \frac{\partial}{\partial z} \left[ D_z(x,y,z,T) \frac{\partial I(x,y,z,t)}{\partial z} \right] - k_{l,v}(x,y,z,T) I(x,y,z,t) \times V(x,y,z,t) \tag{4}
\]

\[
\frac{\partial V(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_x(x,y,z,T) \frac{\partial V(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_y(x,y,z,T) \frac{\partial V(x,y,z,t)}{\partial y} \right] - k_{v,v}(x,y,z,T) \times \]

\[
\times V^2(x,y,z,t) + \frac{\partial}{\partial z} \left[ D_z(x,y,z,T) \frac{\partial V(x,y,z,t)}{\partial z} \right] - k_{v,v}(x,y,z,T) I(x,y,z,t) \times V(x,y,z,t). \tag{5}
\]

Boundary and initial conditions for these equations are

\[
\frac{\partial \rho(x,y,z,t)}{\partial x} \bigg|_{x=0} = 0, \quad \frac{\partial \rho(x,y,z,t)}{\partial x} \bigg|_{x=L_x} = 0, \quad \frac{\partial \rho(x,y,z,t)}{\partial y} \bigg|_{y=0} = 0, \quad \frac{\partial \rho(x,y,z,t)}{\partial y} \bigg|_{y=L_y} = 0, \quad \frac{\partial \rho(x,y,z,t)}{\partial z} \bigg|_{z=0} = 0, \quad \frac{\partial \rho(x,y,z,t)}{\partial z} \bigg|_{z=L_z} = 0, \quad \rho(x,y,z,0) = f_p(x,y,z). \tag{6}
\]

Here \( \rho = I, V \). We denote spatio-temporal distribution of concentration of radiation interstitials as \( I(x,y,z,t) \). Dependences of the diffusion coefficients of point radiation defects on coordinate and temperature have been denoted as \( D_d(x,y,z,T) \). The quadruple on concentration terms of Eqs. (4) describes generation divacancies and diinterstitials. Parameter of recombination of point radiation defects and parameters of generation of simplest complexes of point radiation defects have been denoted as the following functions \( k_{l,v}(x,y,z,T) \), \( k_{l,v}(x,y,z,T) \) and \( k_{v,v}(x,y,z,T) \), respectively.

Now let us calculate distributions of concentrations of divacancies \( \Phi_l(x,y,z,t) \) and diinterstitials \( \Phi_v(x,y,z,t) \) in space and time by solving the following system of equations [15,16]

\[
\frac{\partial \Phi_l(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_{\Phi_l}(x,y,z,T) \frac{\partial \Phi_l(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{\Phi_l}(x,y,z,T) \frac{\partial \Phi_l(x,y,z,t)}{\partial y} \right] + \]

\[
+ \frac{\partial}{\partial z} \left[ D_{\Phi_l}(x,y,z,T) \frac{\partial \Phi_l(x,y,z,t)}{\partial z} \right] + k_{l,l}(x,y,z,T) I^2(x,y,z,t) - k_{l,v}(x,y,z,T) I(x,y,z,t) \tag{7}
\]

\[
\frac{\partial \Phi_v(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_{\Phi_v}(x,y,z,T) \frac{\partial \Phi_v(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{\Phi_v}(x,y,z,T) \frac{\partial \Phi_v(x,y,z,t)}{\partial y} \right] + \]

\[
+ \frac{\partial}{\partial z} \left[ D_{\Phi_v}(x,y,z,T) \frac{\partial \Phi_v(x,y,z,t)}{\partial z} \right] + k_{v,v}(x,y,z,T) V^2(x,y,z,t) - k_{v,l}(x,y,z,T) I(x,y,z,t). \tag{8}
\]
Boundary and initial conditions for these equations are

\[
\frac{\partial \Phi_r (x, y, z, t)}{\partial x} \bigg|_{t=0} = 0, \quad \frac{\partial \Phi_r (x, y, z, t)}{\partial x} \bigg|_{t=t_x} = 0, \quad \frac{\partial \Phi_r (x, y, z, t)}{\partial y} \bigg|_{y=0} = 0, \quad \frac{\partial \Phi_r (x, y, z, t)}{\partial y} \bigg|_{y=t_y} = 0, \quad \frac{\partial \Phi_r (x, y, z, t)}{\partial z} \bigg|_{z=0} = 0, \quad \Phi_r (x, y, z, 0) = f_{\Phi_r} (x, y, z), \quad \Phi_r (x, y, z, 0) = f_{\Phi_r} (x, y, z). \quad (7)
\]

The functions \( D_{\Phi}(x, y, z, T) \) describe dependences of the diffusion coefficients of the above complexes of radiation defects on coordinate and temperature. The functions \( k(x, y, z, T) \) and \( k_i(x, y, z, T) \) describe the parameters of decay of these complexes on coordinate and temperature.

To determine spatio-temporal distribution of concentration of dopant we transform the Eq.(1) to the following integro-differential form

\[
\int_{L_x}^{x} \int_{L_y}^{y} \int_{L_z}^{z} C(u, v, w, t) \, d w \, d v \, d u = \int_{L_x}^{x} \int_{L_y}^{y} \int_{L_z}^{z} D_L (x, v, w, t) \left[ 1 + \xi_1 \frac{V(x, v, w, \tau)}{V^*} + \xi_2 \frac{V^2(x, v, w, \tau)}{(V^*)^2} \right] \times
\]

\[
\times \left[ 1 + \xi_1 \frac{C^r (x, v, w, \tau)}{P^r (x, v, w, T)} \frac{\partial C(x, v, w, \tau)}{\partial x} \right. \left. \frac{y z}{L_x L_z} + \int_{L_x}^{x} \int_{L_y}^{y} \int_{L_z}^{z} D_L (u, y, w, T) \left[ 1 + \xi_1 \frac{C^r (u, y, w, \tau)}{P^r (x, y, z, T)} \frac{y z}{L_x L_z} \right] \right] \frac{y z}{L_y L_z} + \int_{L_x}^{x} \int_{L_y}^{y} \int_{L_z}^{z} D_L (u, v, z, T) \times
\]

\[
\left[ 1 + \xi_1 \frac{V(u, y, w, \tau)}{V^*} + \xi_2 \frac{V^2(u, y, w, \tau)}{(V^*)^2} \right] \frac{\partial C(u, y, w, \tau)}{\partial y} \left. \frac{y z}{L_x L_z} \right] + \int_{L_x}^{x} \int_{L_y}^{y} \int_{L_z}^{z} D_L (u, v, z, T) \times
\]

\[
\left[ 1 + \xi_1 \frac{C^r (u, v, z, \tau)}{P^r (x, y, z, T)} \frac{y z}{L_x L_z} \right] \frac{y z}{L_y L_z} + \int_{L_x}^{x} \int_{L_y}^{y} \int_{L_z}^{z} D_L (u, v, w, T) \right. \left. \frac{y z}{L_x L_z} \right] \frac{y z}{L_y L_z} \int_{L_x}^{x} \int_{L_y}^{y} \int_{L_z}^{z} f (u, v, w) \, d w \, d v \, d u. \quad (1a)
\]

Now let us determine solution of Eq.(1a) by Bubnov-Galerkin approach [17]. To use the approach we consider solution of the Eq.(1a) as the following series

\[
C_0 (x, y, z, t) = \sum_{n=0}^{N} a_{nc} c_n (x) c_n (y) c_n (z) e_{nc} (t).
\]

Here \( e_{nc} (t) = \exp \left[ -\pi n^2 D_{nc} t (L_x^2 + L_y^2 + L_z^2) \right], c_n (\chi) = \cos (\pi n \chi / L_x) \). Number of terms \( N \) in the series is finite. The above series is almost the same with solution of linear Eq.(1) (i.e. for \( \xi = 0 \)) and averaged dopant diffusion coefficient \( D_0 \). Substitution of the series into Eq.(1a) leads to the following result

\[
\frac{x y z}{\pi} \sum_{n=0}^{N} a_{nc} s_n (x) s_n (y) s_n (z) e_{nc} (t) = -\frac{y z}{L_x L_z} \int_{L_x}^{x} \int_{L_y}^{y} \int_{L_z}^{z} \left[ 1 + \left( \sum_{n=0}^{N} a_{nc} c_n (x) c_n (y) c_n (z) e_{nc} (t) \right) \right] \times
\]

\[
\times \left[ 1 + \xi_1 \frac{V(x, v, w, \tau)}{V^*} + \xi_2 \frac{V^2(x, v, w, \tau)}{(V^*)^2} \right] D_L (x, v, w, T) \sum_{n=1}^{N} a_{nc} s_n (x) c_n (v) \times
\]
\[\times n c_n(w) e_{ac}(\tau) d \tau - \frac{x z}{L_1 L_2 L_3} \int \int \int \left\{ 1 + \left[ \sum_{m=1}^{N} a_{ac} c_m(u) c_n(y) c_m(w) e_{ac}(\tau) \right] \right\} \times \]

\[\times \frac{\xi}{P^p(u,y,w,T)} \left\{ 1 + \xi_1 \frac{V(x,y,z,\tau)}{V^*} + \xi_2 \frac{V^2(x,y,z,\tau)}{(V^*)^2} \right\} \sum_{n} a_{ac} c_n(u) c_n(y) c_n(z) e_{ac}(\tau) d \tau \times \]

\[\times \frac{\xi}{P^p(u,y,w,T)} \left\{ 1 + \xi_1 \frac{V(x,y,z,\tau)}{V^*} + \xi_2 \frac{V^2(x,y,z,\tau)}{(V^*)^2} \right\} \sum_{n} a_{ac} c_n(u) c_n(y) c_n(z) e_{ac}(\tau) \times \]

\[\times \frac{\xi}{P^p(u,y,w,T)} \left\{ 1 + \xi_1 \frac{V(x,y,z,\tau)}{V^*} + \xi_2 \frac{V^2(x,y,z,\tau)}{(V^*)^2} \right\} \sum_{n} a_{ac} c_n(u) c_n(y) c_n(z) e_{ac}(\tau) \times \]

where \( s_n(\xi) = \sin(\pi n \xi L_3) \). We used condition of orthogonality to determine coefficients \( a_n \) in the considered series. The coefficients \( a_n \) could be calculated for any quantity of terms \( N \). In the common case the relations could be written as

\[-\frac{L_1 L_2 L_3}{\pi^3} \sum_{n=1}^{N} a_{ac} e_{ac}(\tau) = -\frac{L_1 L_2 L_3}{\pi^3} \int \int \int \left\{ 1 + \left[ \sum_{m=1}^{N} a_{ac} c_m(x) c_n(y) c_m(z) e_{ac}(\tau) \right] \right\} \times \]

\[\times \frac{\xi}{P^p(x,y,z,T)} \left\{ 1 + \xi_1 \frac{V(x,y,z,\tau)}{V^*} + \xi_2 \frac{V^2(x,y,z,\tau)}{(V^*)^2} \right\} \sum_{n} a_{ac} c_n(x) c_n(y) c_n(z) e_{ac}(\tau) \times \]

\[\times \left\{ y s_n(y) + \frac{L_1}{\pi n} c_n(y) - 1 \right\} \left\{ z s_n(z) + \frac{L_1}{\pi n} c_n(z) - 1 \right\} d x d y d z \times \]

\[\times D_L(x,y,z,T) \left\{ 1 + \left[ \sum_{m=1}^{N} a_{ac} c_m(x) c_n(y) c_m(z) e_{ac}(\tau) \right] \right\} \frac{\xi}{P^p(x,y,z,T)} \left\{ 1 + \xi_1 \frac{V(x,y,z,\tau)}{V^*} + \xi_2 \frac{V^2(x,y,z,\tau)}{(V^*)^2} \right\} \sum_{n} a_{ac} c_n(x) c_n(y) c_n(z) e_{ac}(\tau) \times \]

\[\times \frac{\xi}{P^p(x,y,z,T)} \left\{ 1 + \xi_1 \frac{V(x,y,z,\tau)}{V^*} + \xi_2 \frac{V^2(x,y,z,\tau)}{(V^*)^2} \right\} \sum_{n} a_{ac} c_n(x) c_n(y) c_n(z) e_{ac}(\tau) \times \]

\[\times \frac{\xi}{P^p(x,y,z,T)} \left\{ 1 + \xi_1 \frac{V(x,y,z,\tau)}{V^*} + \xi_2 \frac{V^2(x,y,z,\tau)}{(V^*)^2} \right\} \sum_{n} a_{ac} c_n(x) c_n(y) c_n(z) e_{ac}(\tau) \times \]
\[
\times \left\{ y s_n(y) + \frac{L_n}{\pi n} [c_n(y) - 1] \right\} e_{ac}(\tau)d\tau dy dx d\tau + \sum_{s=0}^{N} \left\{ x s_n(x) + \frac{L_n}{\pi n} [c_n(x) - 1] \right\} \times \\
\times \left\{ y s_n(y) + \frac{L_n}{\pi n} [c_n(y) - 1] \right\} f(x, y, z)d\tau dy dx .
\]

As an example for \( \gamma = 0 \) we obtain
\[
\times \left\{ y s_n(y) + \frac{L_n}{\pi n} [c_n(y) - 1] \right\} \times \\
\times \left\{ y s_n(y) + \frac{L_n}{\pi n} [c_n(y) - 1] \right\} f(x, y, z)d\tau dy dx.
\]

For \( \gamma = 1 \) one can obtain the following relation to determine required parameters
\[
a_ac = -\frac{\beta_n}{2\alpha_n} \pm \frac{\alpha_c}{2\alpha_n} \sqrt{\beta_n^2 + 4\alpha_c \left\{ s_n(x) \right\} s_n(y) \left\{ s_n(z) \right\} f(x, y, z)d\tau dy dx} ,
\]

where \( \alpha_c = \frac{\xi L_n L_c}{2\pi n} \left\{ e_{ac}(\tau) \right\} s_n(x) \left\{ s_n(y) \right\} \left\{ s_n(z) \right\} \left[ 1 + \xi_1 \frac{V(x, y, z, \tau)}{V^*} + \xi_2 \frac{V^*(x, y, z, \tau)}{V^*^3} \right] \times \\
+ \xi_1 \frac{V(x, y, z, \tau)}{V^*} \right\} d\tau dy dx d\tau \}
\]

\[
-\frac{\xi L_n L_c}{2\pi n} \left\{ e_{ac}(\tau) \right\} s_n(x) \left\{ s_n(y) \right\} \left\{ s_n(z) \right\} \left[ 1 + \xi_1 \frac{V(x, y, z, \tau)}{V^*} + \xi_2 \frac{V^*(x, y, z, \tau)}{V^*^3} \right] \times
\]
\[ \frac{D_z(x,y,z,T)}{P(x,y,z,T)} \left\{ y_s(y) + \frac{L_x}{\pi n} \cdot \frac{L_z}{\pi n} - \frac{L_z}{\pi n} \cdot \frac{L_z}{\pi n} \right\} \left\{ \int z_s(z) + \frac{L_x}{\pi n} \cdot \frac{L_z}{\pi n} - \frac{L_z}{\pi n} \cdot \frac{L_z}{\pi n} \right\} \right\} d z d y d x d \tau + \frac{\xi_z}{2 \pi n} \times \]

\[ \frac{D_z(x,y,z,T)}{P(x,y,z,T)} \left[ 1 + \xi_z \frac{V(x,y,z,\tau)}{V'} + \frac{\xi_z V^2(x,y,z,\tau)}{(V')^2} \right] d z \right\} d y d x d \tau + \frac{\xi_z}{2 \pi n} \times \]

\[ \frac{\xi_z}{2 \pi n} \times \frac{D_z(x,y,z,T)}{P(x,y,z,T)} \left\{ y_s(y) + \frac{L_x}{\pi n} \cdot \frac{L_z}{\pi n} - \frac{L_z}{\pi n} \cdot \frac{L_z}{\pi n} \right\} \left\{ \int z_s(z) + \frac{L_x}{\pi n} \cdot \frac{L_z}{\pi n} - \frac{L_z}{\pi n} \cdot \frac{L_z}{\pi n} \right\} \right\} d z d y d x d \tau + \frac{\xi_z}{2 \pi n} \times \]

\[ \frac{\xi_z}{2 \pi n} \times \frac{D_z(x,y,z,T)}{P(x,y,z,T)} \left\{ y_s(y) + \frac{L_x}{\pi n} \cdot \frac{L_z}{\pi n} - \frac{L_z}{\pi n} \cdot \frac{L_z}{\pi n} \right\} \left\{ \int z_s(z) + \frac{L_x}{\pi n} \cdot \frac{L_z}{\pi n} - \frac{L_z}{\pi n} \cdot \frac{L_z}{\pi n} \right\} \right\} d z d y d x d \tau + \frac{\xi_z}{2 \pi n} \times \]

\[ \frac{\xi_z}{2 \pi n} \times \frac{D_z(x,y,z,T)}{P(x,y,z,T)} \left\{ y_s(y) + \frac{L_x}{\pi n} \cdot \frac{L_z}{\pi n} - \frac{L_z}{\pi n} \cdot \frac{L_z}{\pi n} \right\} \left\{ \int z_s(z) + \frac{L_x}{\pi n} \cdot \frac{L_z}{\pi n} - \frac{L_z}{\pi n} \cdot \frac{L_z}{\pi n} \right\} \right\} d z d y d x d \tau + \frac{\xi_z}{2 \pi n} \times \]

\[ \frac{\xi_z}{2 \pi n} \times \frac{D_z(x,y,z,T)}{P(x,y,z,T)} \left\{ y_s(y) + \frac{L_x}{\pi n} \cdot \frac{L_z}{\pi n} - \frac{L_z}{\pi n} \cdot \frac{L_z}{\pi n} \right\} \left\{ \int z_s(z) + \frac{L_x}{\pi n} \cdot \frac{L_z}{\pi n} - \frac{L_z}{\pi n} \cdot \frac{L_z}{\pi n} \right\} \right\} d z d y d x d \tau + \frac{\xi_z}{2 \pi n} \times \]

The same approach could be used for calculation parameters \( a_n \) for different values of parameter \( \gamma \). However the relations are bulky and will not be presented in the paper. Advantage of the approach is absent of necessity to join dopant concentration on interfaces of heterostructure. The same Bubnov-Galerkin approach has been used for solution the Eqs. (4). Previously we transform the differential equations to the following integro- differential form

\[ \frac{x y z}{L, L, L} \int \int \int I(u, v, w, \tau) d w d v d u = \frac{x y z}{L, L, L} \int \int \int D_z(x, y, z, T) \int k(u, v, w, \tau) I(u, v, w, \tau) d w d v d \tau + \]

\[ \frac{x y z}{L, L, L} \int \int \int D_z(u, y, z, T) \int k(u, v, w, \tau) I(u, v, w, \tau) d w d u d \tau - \frac{x y z}{L, L, L} \int \int \int D_z(x, y, z, T) \int k(u, v, w, \tau) I(u, v, w, \tau) d w d v d \tau. \]
\[ \times V(u,v,w,t) \text{ d} w \text{ d} v \text{ d} u + \frac{xy}{L_x L_y L_z} \int \int \int \int \frac{\partial I(u,v,z,\tau)}{\partial z} D_{\tau}(u,v,z,T) \text{ d} v \text{ d} u \text{ d} \tau - \frac{xyz}{L_x L_y L_z} \times \]

\[ \times \int \int \int k_{I,t}(u,v,w,T) I^2(u,v,w,t) \text{ d} w \text{ d} v \text{ d} u + \frac{xyz}{L_x L_y L_z} \int \int \int f_t(u,v,w) \text{ d} w \text{ d} v \text{ d} u \]  

(4a)

\[ \frac{xyz}{L_x L_y L_z} \int \int \int V(u,v,w,t) \text{ d} w \text{ d} v \text{ d} u = \frac{yz}{L_y L_z} \int \int \int D_v(x,v,w,T) \frac{\partial V(x,v,w,\tau)}{\partial x} \text{ d} w \text{ d} v \text{ d} \tau + \]

\[ + \frac{xyz}{L_x L_y L_z} \int \int \int D_v(u,y,w,T) \frac{\partial V(x,y,w,\tau)}{\partial y} \text{ d} w \text{ d} u \text{ d} \tau + \frac{xyz}{L_x L_y L_z} \int \int \int \frac{\partial V(x,v,w,\tau)}{\partial z} \times \]

\[ \times D_v(u,v,z,T) \text{ d} v \text{ d} u \text{ d} \tau - \frac{xyz}{L_x L_y L_z} \int \int \int k_{I,v}(u,v,w,T) I(u,v,w,t) V(u,v,w,t) \text{ d} w \text{ d} v \text{ d} u - \]

\[ - \frac{xyz}{L_x L_y L_z} \int \int \int k_{I,v}(u,v,w,T) V^2(u,v,w,t) \text{ d} w \text{ d} v \text{ d} u + \frac{xyz}{L_x L_y L_z} \int \int \int \frac{\partial V(u,v,w,\tau)}{\partial z} \times \]

\[ \times D_v(u,v,w,T) \text{ d} v \text{ d} u \text{ d} \tau - \frac{xyz}{L_x L_y L_z} \int \int \int k_{I,v}(u,v,w,T) I(u,v,w,t) V(u,v,w,t) \text{ d} w \text{ d} v \text{ d} u . \]

We determine spatio-temporal distributions of concentrations of point defects as the same series

\[ \rho_o(x,y,z,t) = \sum_{n=0}^{N} a_{n \rho} c_n(x) c_n(y) c_n(z) e_{n \rho}(t) . \]

Parameters \( a_{n \rho} \) should be determined in future. Substitution of the series into Eqs.(4a) leads to the following results

\[ \frac{xyz}{\pi} \sum_{n=1}^{\infty} a_n \int s_n(x) s_n(y) s_n(z) e_n(t) \text{ d} w \text{ d} v \times \]

\[ \times e_n(t) \text{ d} \tau \times \sum_{n=1}^{\infty} a_n \int s_n(y) e_n(t) \int c_n(z) D_v(x,v,w,T) \text{ d} w \text{ d} v \text{ d} u \times \]

\[ \times e_n(t) \int c_n(v) c_n(w) c_n(t) \text{ d} v \text{ d} w \text{ d} u \]

\[ \times e_n(t) \int c_n(u) c_n(v) c_n(w) c_n(t) k_{I,v}(u,v,T) \text{ d} w \text{ d} v \text{ d} u + \int \int \int f_t(u,v,w) \text{ d} w \text{ d} v \text{ d} u \times \]

\[ \times e_n(t) \sum_{n=1}^{\infty} a_n \int c_n(u) c_n(v) c_n(w) c_n(t) k_{I,v}(u,v,T) \text{ d} w \text{ d} v \text{ d} u \times \]

\[ \times \frac{xyz}{L_x L_y L_z} \int \int \int \int \frac{\partial}{\partial z} D_v(x,v,w,T) \text{ d} w \text{ d} v \text{ d} u \times \]

\[ \times \frac{xyz}{L_x L_y L_z} \int \int \int \int k_{I,v}(u,v,w,T) I(u,v,w,t) V(u,v,w,t) \text{ d} w \text{ d} v \text{ d} u . \]
\[ \times e_{av}(\tau) d\tau \mid s_a(x) - \frac{x\pi}{L_1 L_2 L_3} \sum_{v=0}^{N} a_{av}(y) c_v(x) c_v(z) D_v(u, y, w, T) dwdvd\tau - \frac{y\pi}{L_1 L_2 L_3} \sum_{v=0}^{N} a_{av}(z) c_v(x) c_v(y) D_v(u, v, z, T) dwdvd\tau - \frac{z\pi}{L_1 L_2 L_3} \sum_{v=0}^{N} a_{av}(u) c_v(y) c_v(z) D_v(u, v, w, T) dwdvd\tau \times \]

\[ \times \left[ \sum_{v=0}^{N} a_{av} c_v(u) c_v(v) e_{av}(t) \right]^2 dwdvd\tau \]

\[ \times \frac{x\pi}{L_1 L_2 L_3} \sum_{v=0}^{N} a_{av}(v) c_v(v) e_{av}(t) k_{v, v}(u, v, w, T) dwdvd\tau \times \]

\[ \times yz/L_1 L_2 L_3. \]

We used orthogonality conditions of functions of the considered series framework the heterostructure to calculate coefficients \( a_{\nu p} \). The coefficients \( a_{\nu} \) could be calculated for any quantity of terms \( N \). In the common case equations for the required coefficients could be written as

\[ \sum_{n=0}^{N} \int \int \int \sum_{v=0}^{N} a_{av} c_v(u) c_v(v) e_{av}(t) dwdvd\tau \]

\[ \times \frac{x\pi}{L_1 L_2 L_3} \sum_{v=0}^{N} a_{av}(v) c_v(v) e_{av}(t) k_{v, v}(u, v, w, T) dwdvd\tau \times \]

\[ \times xz/L_1 L_2 L_3. \]
\[ + y s_{x} (2 y) + \frac{L_{x}}{2 \pi n} [e_{x} (2 y) - 1] \int_{0}^{L_{y}} \int_{0}^{L_{z}} f_{x, y} (x, y, z, T) \left[ L_{z} + z s_{z} (2 z) + \frac{L_{z}}{2 \pi n} [e_{z} (2 z) - 1] \right] d z \times \]
\[ \times d y d x + \sum_{n=1}^{N} \left[ x s_{x} (x) + \frac{L_{x}}{\pi n} [e_{x} (x) - 1] \right] \int_{0}^{L_{y}} \int_{0}^{L_{z}} f_{x, y} (x, y, z, T) \left[ L_{z} + z s_{z} (2 z) + \frac{L_{z}}{2 \pi n} [e_{z} (2 z) - 1] \right] d z \times \]
\[ \times \left\{ L_{z} + z s_{z} (2 z) + \frac{L_{z}}{2 \pi n} [e_{z} (2 z) - 1] \right\} d z d y d x \]

\[ - \frac{L_{x}^{2} L_{y}^{2} L_{z}^{2}}{\pi^{5}} \sum_{n=1}^{N} \frac{a_{n}^{y}}{n^3} e_{n} (t) = - \frac{1}{2 \pi L_{x}} \sum_{n=1}^{N} \frac{a_{n}^{y}}{n^2} \int_{0}^{L_{x}} \int_{0}^{L_{y}} \left[ L_{x} + y s_{x} (2 y) + \frac{L_{x}}{2 \pi n} [e_{x} (2 y) - 1] \right] \times \]
\[ \times \int_{0}^{L_{z}} \int_{0}^{L_{y}} \int_{0}^{L_{z}} f_{x, y, z} (x, y, z, T) \left[ L_{z} + z s_{z} (2 z) + \frac{L_{z}}{2 \pi n} [e_{z} (2 z) - 1] \right] d z \left[ 1 - e_{n} (2 y) \right] \times \]
\[ \times d y d x d e_{n} (t) d \tau - \frac{1}{2 \pi L_{x}} \sum_{n=1}^{N} \frac{a_{n}^{y}}{n^2} \int_{0}^{L_{x}} \int_{0}^{L_{y}} \left[ L_{x} + x s_{x} (2 x) + \frac{L_{x}}{2 \pi n} [e_{x} (2 x) - 1] \right] \times \]
\[ \times \int_{0}^{L_{z}} \int_{0}^{L_{y}} \int_{0}^{L_{z}} f_{x, y, z} (x, y, z, T) \left[ L_{z} + z s_{z} (2 z) + \frac{L_{z}}{2 \pi n} [e_{z} (2 z) - 1] \right] d z \left[ 1 - e_{n} (2 x) \right] \times \]
\[ \times \left\{ L_{x} + \frac{L_{x}}{2 \pi n} [e_{x} (2 x) - 1] \right\} \int_{0}^{L_{x}} d x \int_{0}^{L_{z}} \int_{0}^{L_{y}} \int_{0}^{L_{z}} f_{x, y, z} (x, y, z, T) \left[ L_{z} + z s_{z} (2 z) + \frac{L_{z}}{2 \pi n} [e_{z} (2 z) - 1] \right] d z \times \]
\[ \times d y d x + \sum_{n=1}^{N} \left[ x s_{x} (x) + \frac{L_{x}}{\pi n} [e_{x} (x) - 1] \right] \int_{0}^{L_{y}} \int_{0}^{L_{z}} f_{x, y} (x, y, z, T) \left[ L_{z} + z s_{z} (2 z) + \frac{L_{z}}{2 \pi n} [e_{z} (2 z) - 1] \right] d z \times \]
\[ \times \left\{ L_{z} + z s_{z} (2 z) + \frac{L_{z}}{2 \pi n} [e_{z} (2 z) - 1] \right\} d z d y d x. \]
In the final form relations for required parameters could be written as
\[
a_{n l} = \frac{b_1 + A}{4b_4} \pm \frac{(b_1 + A)^2}{4} - 4b_4 \left( y + b_1 y - \chi_{nl} \right), \quad a_{n l} = -\chi_{nl} a_{nl} + \delta_{nl} a_{nl} + \lambda_{nl},
\]
where
\[
\gamma_{np} = e_{np} (2t) \int_0^t k_{np} (x, y, z, T) \left\{ L_x + x s_n (2x) + \frac{L_z}{2\pi n} [c_n (2x) - 1] \right\} dy dz dt,
\]
and time as the following functional series
\[
\Phi = \sum_{n=1}^N a_{n \phi} \left\{ c_n (x) c_n (y) c_n (z) e_{np} (t) \right\}.
\]
Here \( a_{n \phi} \) are the coefficients, which should be determined. Let us previously transform the Eqs. (6) to the following integro-differential form.
Substitution of the previously considered series in the Eqs. (6a) leads to the following form

\[ -\sum_{n=1}^{N} \frac{a_{\phi}(t)}{n} s_n(x) s_n(y) s_n(z) e_{\alpha}(t) = -\sum_{n=1}^{N} \frac{y \pi}{L_L L_L} \sum_{n=1}^{N} n a_{\phi}(t) s_n(x) e_{\alpha}(t) c_n(y) c_n(z) \times \]

\[ \times \int \int \int D_{\phi}(u,v,w,T) d w d v d u = -\sum_{n=1}^{N} \frac{y \pi}{L_L L_L} \sum_{n=1}^{N} n a_{\phi}(t) s_n(x) e_{\alpha}(t) c_n(y) c_n(z) \times \]

\[ \times \int \int \int D_{\phi}(u,v,w,T) d w d v d u \]

\[ -\sum_{n=1}^{N} \frac{a_{\phi}(t)}{n} s_n(x) s_n(y) s_n(z) e_{\alpha}(t) = -\sum_{n=1}^{N} \frac{y \pi}{L_L L_L} \sum_{n=1}^{N} n a_{\phi}(t) s_n(x) e_{\alpha}(t) c_n(y) c_n(z) \times \]

\[ \times \int \int \int D_{\phi}(u,v,w,T) d w d v d u \]

\[ -\sum_{n=1}^{N} \frac{a_{\phi}(t)}{n} s_n(x) s_n(y) s_n(z) e_{\alpha}(t) = -\sum_{n=1}^{N} \frac{y \pi}{L_L L_L} \sum_{n=1}^{N} n a_{\phi}(t) s_n(x) e_{\alpha}(t) c_n(y) c_n(z) \times \]
\[\times D_{\psi_v} (x, v, w, T) \frac{d w d v d \tau}{2} = \frac{x z \pi}{L_z L_y L_x} \sum_{n=1}^{\infty} \sum_{i=0}^{\infty} \int \int c_n(u) c_n(w) D_{\psi_v} (u, v, w, T) \frac{d w d u d \tau}{2}\times\]

\[\times a_{\psi_v} s_n(y) e_{\psi_v}(t) - \frac{x y z}{L_z L_y L_x} \sum_{n=1}^{\infty} \sum_{i=0}^{\infty} s_n(z) e_{\psi_v}(t) \frac{d w d v d u}{2}\times\]

\[\times \sum_{n=1}^{\infty} \sum_{i=0}^{\infty} f_v (u, v, w) d w d v d u \times\]

\[\times \frac{x y z}{L_z L_y L_x} \int \int k_v (u, v, w, T) V (u, v, w, \tau) d w d v d u .\]

We used orthogonality condition of functions of the considered series framework the heterostructure to calculate coefficients \(a_{\psi_v}\). The coefficients \(a_{\psi_v}\) could be calculated for any quantity of terms \(N\). In the common case equations for the required coefficients could be written as:

\[-\frac{L_z L_y L_x}{\pi} \sum_{n=1}^{\infty} a_{n \psi_v} e_{n \psi_v}(t) = -\frac{1}{2 \pi L_z} \int \int \left[ [-c_n(2x)] \frac{L_z}{2 \pi n} + \frac{L_y}{2 \pi n} \left[ c_n(2y) - 1 \right] \right] x d y d x e_{n \psi_v}(t) d \tau\times\]

\[\int \int k_v (u, v, w, T) V (u, v, w, \tau) d w d v d u = \sum_{n=1}^{\infty} \sum_{i=0}^{\infty} f_v (u, v, w) d w d v d u \times\]

\[\times \frac{x y z}{L_z L_y L_x} \int \int k_v (u, v, w, T) V (u, v, w, \tau) d w d v d u .\]
In the present paper we analyzed redistribution of infused and implanted dopants in a heterostructure, which have been presented on Fig. 1. The analysis has been done by using relations, calculated in the previous section. First of all we consider situation, when dopant diffusion coefficient in doped area is larger, than in nearest areas. It has been shown, that in this case distribution of concentration of dopant became more compact in comparison with wise versa situation (see Figs. 2 and 3 for diffusion and ion types of doping). In the wise versa situation (when dopant diffusion coefficient in doped area is smaller, than in nearest areas) we obtained spreading of distribution of dopant. In this case outside of doping material one can find higher spreading in comparison with wise versa situation (see Fig. 4).

It should be noted, that properties of layers of multilayer structure varying in space: varying layers with larger and smaller values of the diffusion coefficient. In this situation with account results, which shown on Figs. 2-4, layers of the considered heterostructure with smaller value of...
dopant should have smaller level of doping in comparison with nearest layers to exclude changing of type of doping in the nearest layers. For a more complete doping of each section and at the same to decrease diffusion of dopant into nearest sections it is attracted an interest optimization of annealing of dopant and/or radiation defects. Let us optimize annealing of dopant and/or radiation defects by using recently introduce criterion [18-23]. Framework the criterion we approximate real distribution of concentration of dopant by idealized step-wise distribution of concentration \( \psi(x,y,z) \) (see Figs. 5 and 6 for diffusive or ion types of doping). Farther we determine optimal annealing time by minimization of mean-squared error.

![Fig. 2. Spatial distributions of infused dopant concentration in the considered heterostructure. The considered direction perpendicular to the interface between epitaxial layer substrate. Difference between values of dopant diffusion coefficient in layers of heterostructure increases with increasing of number of curves.](image2)

![Fig. 3. Spatial distributions of infused dopant concentration in the considered heterostructure. Curves 1 and 3 corresponds to annealing time \( \Theta = 0.0048(L^2 + L_y^2 + L_z^2)/D_0 \). Curves 2 and 4 corresponds to annealing time \( \Theta = 0.0057(L^2 + L_y^2 + L_z^2)/D_0 \). Curves 1 and 2 corresponds to homogenous sample. Curves 3 and 4 corresponds to the considered heterostructure. Difference between values of dopant diffusion coefficient in layers of heterostructure increases with increasing of number of curves.](image3)
Difference between values of dopant diffusion coefficient in layers of heterostructure increases with increasing of number of curves

\[ U = \frac{1}{L, L, L} \int \int \int [C(x, y, z, \Theta) - \psi(x, y, z)] \, dz \, dy \, dx. \]  

Dependences of optimal annealing time are shown on Figs. 7 and 8 for diffusion and ion types of doping. It should be noted, that after finishing of ion doping of material it is necessary to anneal radiation defects. In the ideal case after finishing of the annealing dopant achieves nearest interface between materials of heterostructure. If the dopant has no time to achieve the interface, it is practically to additionally anneal the dopant. The Fig. 8 shows dependences of the exactly additional annealing time of implanted dopant. Necessity of annealing of radiation defects leads to smaller values of optimal annealing time for ion doping in comparison with values of optimal annealing time for diffusion type of doping. Using diffusion type of doping did not leads to so large damage in comparison with damage during ion type of doping.
Curves 1-4 are the real distributions of concentrations of dopant for different values of annealing time for increasing of annealing time with increasing of number of curve.

Dependences of optimal annealing time are shown on Figs. 7 and 8 for diffusion and ion types of doping. It should be noted, that after finishing of ion doping of material it is necessary to anneal radiation defects. In the ideal case after finishing of the annealing dopant achieves nearest interface between materials of heterostructure. If the dopant has no time to achieve the interface, it is practicably to additionally anneal the dopant. The Fig. 8 shows dependences of the exactly additional annealing time of implanted dopant. Necessity of annealing of radiation defects leads to smaller values of optimal annealing time for ion doping in comparison with values of optimal annealing time for diffusion type of doping. Using diffusion type of doping did not leads to so large damage in comparison with damage during ion type of doping.

Curve 1 is the dependence of the considered annealing time on dimensionless thickness of epitaxial layer $a/L$ and $\xi=\gamma=0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of the considered annealing time on the parameter $\varepsilon$ for $a/L=1/2$ and $\xi=\gamma=0$. Curve 3 is the dependence of the considered annealing time on the parameter $\xi$ for $a/L=1/2$ and $\varepsilon=\gamma=0$. Curve 4 is the dependence of the considered annealing time on parameter $\gamma$ for $a/L=1/2$ and $\varepsilon=\xi=0$. 
Curve 1 is the dependence of the considered annealing time on dimensionless thickness of epitaxial layer $a/L$ and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of the considered annealing time on the parameter $\varepsilon$ for $a/L=1/2$ and $\xi = \gamma = 0$. Curve 3 is the dependence of the considered annealing time on the parameter $\xi$ for $a/L=1/2$ and $\varepsilon = \gamma = 0$. Curve 4 is the dependence of the considered annealing time on parameter $\gamma$ for $a/L=1/2$ and $\varepsilon = \xi = 0$.

4. CONCLUSIONS

In this paper we introduced an approach to manufacture of field-effect of transistors which gives a possibility to decrease their dimensions. The decreasing based on manufacturing the transistors in a heterostructure with specific configuration, doping of required areas of the heterostructure by diffusion or ion implantation and optimization of annealing of dopant and/or radiation defects. Framework the approach we introduce an approach of additional doping of channel. The additional doping gives us possibility to modify energy band diagram. We also consider an analytical approach to model and optimize technological process.

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