

ON DECREASING OF DIMENSIONS OF FIELD-EFFECT TRANSISTORS WITH SEVERAL SOURCES

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ABSTRACT

We analyzed mass and heat transport during manufacturing field-effect heterotransistors with several sources to decrease their dimensions. Framework the result of manufacturing it is necessary to manufacture heterostructure with specific configuration. After that it is necessary to dope required areas of the heterostructure by diffusion or ion implantation to manufacture the required type of conductivity (p or n). After the doping it is necessary to do optimize annealing. We introduce an analytical approach to prognosis mass and heat transport during technological processes. Using the approach leads to take into account nonlinearity of mass and heat transport and variation in space and time (at one time) physical parameters of these processes

KEYWORDS

Field-effect transistor, transistor with several channels, increasing of compactness of transistors

1. INTRODUCTION

Now several problems of solid state electronic intensively solving. The problems are increasing of density of elements of integrated circuits and at the same time decreasing of dimensions of these elements [1-4], increasing performance [5-7] and increasing reliability [8,9]. Now one can find intensive development of both power electronic devices and logical elements. In this paper we consider an approach to manufacture more compact field-effect heterotransistor with several sources. Framework the approach it is necessary to manufacture a heterostructure. The heterostructure consist of a substrate and an epitaxial layer (see Fig. 1). Several sections have been manufactured into the epitaxial layer. These sections manufactured by using other materials (see Fig. 1). The sections have been doped by diffusion or ion implantation to obtain required type of conductivity (n or p). After the doping one can manufacture a field-effect transistor framework the considered heterostructure so as it is shown on the Fig. 1. The doping should be finished by annealing of dopant and/or radiation defects. The annealing should be optimized. The optimization attracted an interest to manufacture more compact distributions of concentrations of dopant. Framework the paper we formulate conditions to increase compactness and at the same time to increase homogeneity of distribution of concentration of dopant in enriched by the dopant area.

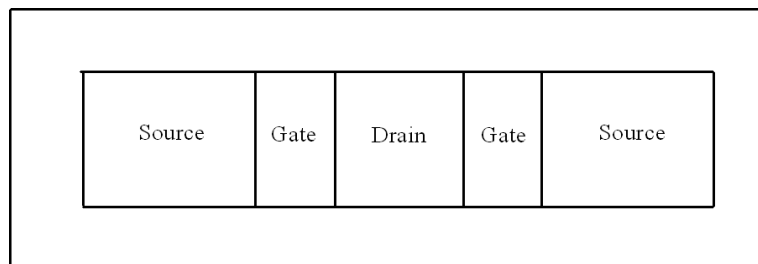


Fig. 1. Structure of a field-effect heterotransistor. Top side of the structure

2. METHOD OF SOLUTION

To solve our aim we determine distribution of concentration of dopant $C(x,y,z,t)$ in space and time. To determine the distribution we solve the following boundary problem

$$\begin{aligned} \frac{\partial C(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x} \left[D \frac{\partial C(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D \frac{\partial C(x,y,z,t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[D \frac{\partial C(x,y,z,t)}{\partial z} \right] \quad (1) \\ \frac{\partial C(x,y,z,t)}{\partial x} \Big|_{x=0} &= 0, \quad \frac{\partial C(x,y,z,t)}{\partial x} \Big|_{x=L_x} = 0, \quad \frac{\partial C(x,y,z,t)}{\partial y} \Big|_{y=0} = 0, \quad \frac{\partial C(x,y,z,t)}{\partial y} \Big|_{y=L_y} = 0, \\ \frac{\partial C(x,y,z,t)}{\partial z} \Big|_{z=0} &= 0, \quad \frac{\partial C(x,y,z,t)}{\partial z} \Big|_{z=L_z} = 0, \quad C(x,y,z,0) = f_C(x,y,z). \quad (2) \end{aligned}$$

Here T is the temperature of annealing; D_C is the dopant diffusion coefficient. Dopant diffusion coefficient takes another value in other materials. Heating and cooling of heterostructure (see Arrhenius law) also leads to changing of value of diffusion coefficient. We consider following approximation of concentrational dependences of dopant diffusion coefficient [10-12]

$$D_C = D_L(x,y,z,T) \left[1 + \xi \frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)} \right] \left[1 + \zeta_1 \frac{V(x,y,z,t)}{V^*} + \zeta_2 \frac{V^2(x,y,z,t)}{(V^*)^2} \right]. \quad (3)$$

Here function $D_L(x,y,z,T)$ describes dependences of dopant diffusion coefficient on coordinate and temperature; function $P(x,y,z,T)$ describes dependences of limit of solubility on coordinate and temperature; parameter $\gamma \in [1,3]$ is integer and depends on properties of materials; function $V(x,y,z,t)$ describes distribution of concentration of radiation vacancies on space and time with equilibrium distribution V^* . Dependence of dopant diffusion coefficient on concentration of dopant has been investigated and described in details in [10]. Diffusion of dopant gives a possibility to dope materials without generation radiation defects. In this situation $\zeta_1 = \zeta_2 = 0$. Ion doping of dopant leads to generation radiation defects. Distributions of concentrations of point radiation defects have been determined by solving the following boundary problem [11,12]

$$\begin{aligned} \frac{\partial I(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_I(x,y,z,T) \frac{\partial I(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_I(x,y,z,T) \frac{\partial I(x,y,z,t)}{\partial y} \right] - k_{I,I}(x,y,z,T) \times \\ &\times I^2(x,y,z,t) + \frac{\partial}{\partial z} \left[D_I(x,y,z,T) \frac{\partial I(x,y,z,t)}{\partial z} \right] - k_{I,V}(x,y,z,T) I(x,y,z,t) V(x,y,z,t) \quad (4) \\ \frac{\partial V(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_V(x,y,z,T) \frac{\partial V(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_V(x,y,z,T) \frac{\partial V(x,y,z,t)}{\partial y} \right] - k_{V,V}(x,y,z,T) \times \\ &\times V^2(x,y,z,t) + \frac{\partial}{\partial z} \left[D_V(x,y,z,T) \frac{\partial V(x,y,z,t)}{\partial z} \right] - k_{I,V}(x,y,z,T) I(x,y,z,t) V(x,y,z,t) \\ \frac{\partial \rho(x,y,z,t)}{\partial x} \Big|_{x=0} &= 0, \quad \frac{\partial \rho(x,y,z,t)}{\partial x} \Big|_{x=L_x} = 0, \quad \frac{\partial \rho(x,y,z,t)}{\partial y} \Big|_{y=0} = 0, \quad \frac{\partial \rho(x,y,z,t)}{\partial y} \Big|_{y=L_y} = 0, \\ \frac{\partial \rho(x,y,z,t)}{\partial z} \Big|_{z=0} &= 0, \quad \frac{\partial \rho(x,y,z,t)}{\partial z} \Big|_{z=L_z} = 0, \quad \rho(x,y,z,0) = f_\rho(x,y,z). \quad (5) \end{aligned}$$

Here $\rho=I,V$; distribution of concentration of radiation interstitials in space and time describes by the function $I(x,y,z,t)$; terms of Eqs.(4) with quadric concentrations of point radiation defects ($V^2(x,y,z,t)$ and $I^2(x,y,z,t)$) correspond to generation of simplest complexes of radiation defects (divacancies and diinterstitials); temperature and spatial dependences of diffusion coefficients of point radiation defects describe by functions $D_\rho(x,y,z,T)$; temperature and spatial dependences of parameter of recombination of point radiation defects describe by function $k_{I,V}(x,y,z,T)$; functions $k_{I,I}(x,y,z,T)$ and $k_{V,V}(x,y,z,T)$ describe spatial and temperature dependences of parameters of generation of simplest complexes of point radiation defects.

Concentrations of divacancies $\Phi_V(x,y,z,t)$ and dinterstitials $\Phi_I(x,y,z,t)$ have been calculated by solution of the following boundary problem [11,12]

$$\begin{aligned} \frac{\partial \Phi_I(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_{\Phi_I}(x,y,z,T) \frac{\partial \Phi_I(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_{\Phi_I}(x,y,z,T) \frac{\partial \Phi_I(x,y,z,t)}{\partial y} \right] + \\ &+ \frac{\partial}{\partial z} \left[D_{\Phi_I}(x,y,z,T) \frac{\partial \Phi_I(x,y,z,t)}{\partial z} \right] + k_{I,I}(x,y,z,T) I^2(x,y,z,t) - k_I(x,y,z,T) I(x,y,z,t) \quad (6) \\ \frac{\partial \Phi_V(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_{\Phi_V}(x,y,z,T) \frac{\partial \Phi_V(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_{\Phi_V}(x,y,z,T) \frac{\partial \Phi_V(x,y,z,t)}{\partial y} \right] + \\ &+ \frac{\partial}{\partial z} \left[D_{\Phi_V}(x,y,z,T) \frac{\partial \Phi_V(x,y,z,t)}{\partial z} \right] + k_{V,V}(x,y,z,T) V^2(x,y,z,t) - k_V(x,y,z,T) V(x,y,z,t) \\ \left. \frac{\partial \Phi_\rho(x,y,z,t)}{\partial x} \right|_{x=0} &= 0, \quad \left. \frac{\partial \Phi_\rho(x,y,z,t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial \Phi_\rho(x,y,z,t)}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial \Phi_\rho(x,y,z,t)}{\partial y} \right|_{y=L_y} = 0, \\ \left. \frac{\partial \Phi_\rho(x,y,z,t)}{\partial z} \right|_{z=0} &= 0, \quad \left. \frac{\partial \Phi_\rho(x,y,z,t)}{\partial z} \right|_{z=L_z} = 0, \quad \Phi_I(x,y,z,0) = f_{\Phi_I}(x,y,z), \quad \Phi_V(x,y,z,0) = f_{\Phi_V}(x,y,z). \quad (7) \end{aligned}$$

Functions $D_{\Phi\rho}(x,y,z,T)$ describe spatial and temperature dependences of diffusion coefficients of simplest complexes of point radiation defects; functions $k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ describe spatial and temperature dependences of parameters of decay of the above complexes.

Now let us consider equivalent integro-differential form of Eq.(1)

$$\begin{aligned} \frac{xyz}{L_x L_y L_z} \int_{L_x}^x \int_{L_y}^y \int_{L_z}^z C(u,v,w,\tau) dw dv du &= \int_0^t \int_{L_y}^y \int_{L_z}^z D_L(x,v,w,T) \left[1 + \zeta_1 \frac{V(x,v,w,\tau)}{V^*} + \zeta_2 \frac{V^2(x,v,w,\tau)}{(V^*)^2} \right] \times \\ &\times \left[1 + \xi \frac{C^\gamma(x,v,w,\tau)}{P^\gamma(x,v,w,T)} \right] \frac{\partial C(x,v,w,\tau)}{\partial x} d\tau \frac{yz}{L_y L_z} + \frac{xz}{L_x L_z} \int_0^t \int_{L_y}^y \int_{L_z}^z D_L(u,y,w,T) \frac{\partial C(u,y,w,\tau)}{\partial y} \times \\ &\times \left[1 + \zeta_1 \frac{V(u,y,w,\tau)}{V^*} + \zeta_2 \frac{V^2(u,y,w,\tau)}{(V^*)^2} \right] \left[1 + \xi \frac{C^\gamma(u,y,w,\tau)}{P^\gamma(x,y,z,T)} \right] d\tau + \frac{xy}{L_x L_y} \int_0^t \int_{L_x}^x \int_{L_z}^z D_L(u,v,z,T) \times \\ &\times \left[1 + \zeta_1 \frac{V(u,v,z,\tau)}{V^*} + \zeta_2 \frac{V^2(u,v,z,\tau)}{(V^*)^2} \right] \left[1 + \xi \frac{C^\gamma(u,v,z,\tau)}{P^\gamma(x,y,z,T)} \right] \frac{\partial C(u,v,z,\tau)}{\partial z} d\tau + \frac{xyz}{L_x L_y L_z} \times \\ &\times \int_{L_x}^x \int_{L_y}^y \int_{L_z}^z f(u,v,w) dw dv du. \quad (1a) \end{aligned}$$

We used the Bubnov-Galerkin approach [13] to calculate solution of the above equation. To use the approach we consider of the Eq.(1a) as the following series

$$C_0(x, y, z, t) = \sum_{n=0}^N a_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t).$$

Here $e_{nC}(t) = \exp[-\pi^2 n^2 D_{0C} t (L_x^{-2} + L_y^{-2} + L_z^{-2})]$, $c_n(\chi) = \cos(\pi n \chi / L_\chi)$. Number of terms N of the above series is finite. The series is almost coincides with solution of Eq.(1) in the linear case (i.e. with $\xi = 0$) and average value of dopant diffusion coefficient D_0 . Framework the approach we substitute the above series into Eq.(1a). After the substitution we obtain

$$\begin{aligned} & \frac{x y z}{\pi^2} \sum_{n=1}^N \frac{a_{nC}}{n^3} s_n(x) s_n(y) s_n(z) e_{nC}(t) = - \int_0^t \int_{L_x}^y \int_{L_z}^z \left\{ 1 + \left[\sum_{n=1}^N a_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(\tau) \right]^\gamma \frac{\xi}{P^\gamma(x, y, z, T)} \right\} \times \\ & \times \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] D_L(x, y, z, T) \sum_{n=1}^N n a_{nC} s_n(x) c_n(y) c_n(z) e_{nC}(\tau) d\tau \frac{y z}{L_y L_z} - \\ & - \frac{x z}{L_x L_z} \int_0^t \int_{L_x}^x \int_{L_z}^z \left[1 + \varsigma_1 \frac{V(u, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(u, y, z, \tau)}{(V^*)^2} \right] \left\{ 1 + \left[\sum_{m=1}^N a_{mC} c_m(u) c_m(y) c_m(z) e_{mC}(\tau) \right]^\gamma \right\} \times \\ & \times \frac{\xi}{P^\gamma(u, y, z, T)} \left\{ D_L(u, y, z, T) \sum_{n=1}^N n a_{nC} c_n(u) s_n(y) c_n(z) e_{nC}(\tau) d\tau - \frac{x y}{L_x L_y} \int_0^t \int_{L_x}^x \int_{L_y}^y D_L(u, v, z, T) \times \right. \\ & \times \left. \left\{ 1 + \frac{\xi}{P^\gamma(u, v, z, T)} \left[\sum_{n=1}^N a_{nC} c_n(u) c_n(v) c_n(z) e_{nC}(\tau) \right]^\gamma \right\} \left[1 + \varsigma_1 \frac{V(u, v, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(u, v, z, \tau)}{(V^*)^2} \right] \right\} \times \\ & \times \sum_{n=1}^N n a_{nC} c_n(u) c_n(v) s_n(z) e_{nC}(\tau) d\tau + \frac{x y z}{L_x L_y L_z} \int_{L_x}^x \int_{L_y}^y \int_{L_z}^z f(u, v, w) d w d v d u. \quad (8) \end{aligned}$$

Here $s_n(\chi) = \sin(\pi n \chi / L_\chi)$. To determine coefficients a_n it is necessary to use orthogonality condition of terms of the above series framework scale of heterostructure. Using the condition leads to the following equations to calculate of coefficients a_n for any quantity of terms N

$$\begin{aligned} & - \frac{L_x^2 L_y^2 L_z^2}{\pi^5} \sum_{n=1}^N \frac{a_{nC}}{n^6} e_{nC}(t) = - \int_0^t \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} D_L(x, y, z, T) \left\{ 1 + \left[\sum_{n=1}^N a_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(\tau) \right]^\gamma \frac{\xi}{P^\gamma(x, y, z, T)} \right\} \times \\ & \times \frac{L_y L_z}{2 \pi^2} \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \sum_{n=1}^N \frac{a_{nC}}{n} s_n(2x) c_n(y) \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \times \\ & \times c_n(z) e_{nC}(\tau) \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} d z d y d x d \tau - \int_0^t \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left\{ \sum_{n=1}^N a_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(\tau) \right\}^\gamma \times \\ & \times \frac{\xi}{P^\gamma(x, y, z, T)} + 1 \left\{ D_L(x, y, z, T) \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \right. \right. \\ & \left. \left. + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \sum_{n=1}^N \frac{a_{nC} L_z}{2 \pi^2 n} \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} c_n(x) s_n(2y) c_n(z) \times \right. \\ & \left. \times e_{nC}(\tau) d z d y d x d \tau - \frac{L_x L_y}{2 \pi^2} \int_0^t \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left\{ 1 + \left[\sum_{n=1}^N a_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(\tau) \right]^\gamma \frac{\xi}{P^\gamma(x, y, z, T)} \right\} \times \right. \end{aligned}$$

$$\begin{aligned} & \times D_L(x, y, z, T) \left[1 + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} \right] \sum_{n=1}^N \frac{a_{nC}}{n} c_n(x) \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \times \\ & \times c_n(y) s_n(z) \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} e_{nC}(\tau) d z d y d x d \tau + \sum_{n=1}^N \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \times \\ & \times \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} f(x, y, z) d z d y d x . \quad (9) \end{aligned}$$

Now we consider several examples. For $\gamma=0$ we obtain

$$\begin{aligned} a_{nC} &= \int_0^{L_x} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} f(x, y, z) d z d y \left\{ x s_n(x) + \frac{L_x}{\pi n} \times \right. \\ & \times [c_n(x) - 1] \Big\} d x \left(\frac{n}{2} \int_0^{L_x} s_n(2x) \int_0^{L_y} c_n(y) \int_0^{L_z} c_n(z) D_L(x, y, z, T) \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} \left\{ y s_n(y) + \right. \right. \\ & \left. \left. + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \left[1 + \frac{\xi}{P^\gamma(x, y, z, T)} \right] c_n(z) d z d y d x \times \right. \\ & \times e_{nC}(\tau) d \tau + \int_0^{L_x} e_{nC}(\tau) \int_0^{L_x} c_n(x) \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} s_n(2y) \int_0^{L_z} c_n(z) \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} \times \\ & \times D_L(x, y, z, T) \left[1 + \frac{\xi}{P^\gamma(x, y, z, T)} \right] \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] d z d y d x d \tau + \\ & \left. + \int_0^{L_x} e_{nC}(\tau) \int_0^{L_x} c_n(x) \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} c_n(y) \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} D_L(x, y, z, T) \times \right. \\ & \times s_n(2z) \left[1 + \frac{\xi}{P^\gamma(x, y, z, T)} \right] \left[1 + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} \right] d z d y d x d \tau \left. \right) - \frac{L_x^2 L_y^2 L_z^2}{\pi^5 n^6} \Big)^{-1}. \quad (10) \end{aligned}$$

For $\gamma=1$ calculation of parameters a_n leads to the following results

$$a_{nC} = -\frac{\beta_n}{2\alpha_n} \pm \sqrt{\beta_n^2 + 4\alpha_n \int_0^{L_x} c_n(x) \int_0^{L_y} c_n(y) \int_0^{L_z} c_n(z) f(x, y, z) d z d y d x} . \quad (11)$$

$$\begin{aligned} \text{Here } \alpha_n &= \frac{\xi L_x L_z}{2\pi^2 n} \int_0^{L_x} s_n(2x) \int_0^{L_y} c_n(y) \int_0^{L_z} \frac{D_L(x, y, z, T)}{P(x, y, z, T)} \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \\ & \times c_n(z) \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} d z d y d x e_{nC}(\tau) d \tau + \frac{\xi L_x L_z}{2\pi^2 n} \times \\ & \times \int_0^{L_x} e_{nC}(\tau) \int_0^{L_x} c_n(x) \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} s_n(2y) \int_0^{L_z} \frac{D_L(x, y, z, T)}{P(x, y, z, T)} \left\{ z s_n(z) - \frac{L_z}{\pi n} [c_n(z) - 1] \right\} \times \\ & \times c_n(z) \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] d z d y d x d \tau + \frac{\xi L_x L_y}{2\pi^2 n} \int_0^{L_x} e_{nC}(\tau) \int_0^{L_x} c_n(x) \int_0^{L_y} c_n(y) \times \end{aligned}$$

$$\begin{aligned}
 & \times \int_0^{L_z} s_n(2z) \frac{D_L(x, y, z, T)}{P(x, y, z, T)} \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \left\{ s_n(y) \times \right. \\
 & \times y + \frac{L_y}{\pi n} [c_n(y) - 1] \left. \right\} d z d y d x d \tau, \beta_n = \frac{L_y L_z}{2n \pi^2} \int_0^t e_{nC}(\tau) \int_0^{L_x} s_n(2x) \int_0^{L_y} c_n(y) \int_0^{L_z} c_n(z) D_L(x, y, z, T) \times \\
 & \times \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} d z \left\{ \frac{L_y}{\pi n} [c_n(y) - 1] + y s_n(y) \right\} \times \\
 & \times d y d x d \tau + \frac{L_x L_z}{2n \pi^2} \int_0^t e_{nC}(\tau) \int_0^{L_x} c_n(x) \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} s_n(2y) \int_0^{L_z} c_n(z) \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \right. \\
 & \left. + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] D_L(x, y, z, T) \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} d z d y d x d \tau + \frac{L_x L_y}{2n \pi^2} \int_0^t e_{nC}(\tau) \times \\
 & \times \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \\
 & \times s_n(2z) D_L(x, y, z, T) d z c_n(y) d y c_n(x) d x d \tau - L_x^2 L_y^2 e_{nC}(t) / \pi^5 n^6.
 \end{aligned}$$

It could be used the same approach to calculate values of parameters a_n for larger values of the parameter γ . However the relations became more bulky and will not be present in the paper. The considered approach gives a possibility to calculate distributions of concentrations of dopant and radiation defects without joining of the above concentration on interfaces of the considered heterostructure.

We solved equations of the system (4) by using Bubnov-Galerkin approach. To use the approach we previously transform the differential equations to the following integro-differential form

$$\begin{aligned}
 & \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z I(u, v, w, t) d w d v d u = \frac{y z}{L_y L_z} \int_0^t \int_0^y \int_0^z D_I(x, v, w, T) \frac{\partial I(x, v, w, \tau)}{\partial x} d w d v d \tau + \frac{x z}{L_x L_z} \times \\
 & \times \int_0^t \int_0^x \int_0^z D_I(u, y, w, T) \frac{\partial I(u, y, w, \tau)}{\partial x} d w d u d \tau + \frac{x y}{L_x L_y} \int_0^t \int_0^x \int_0^y D_I(u, v, z, T) \frac{\partial I(u, v, z, \tau)}{\partial z} d v d u d \tau - \\
 & - \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z k_{I,V}(u, v, w, T) I(u, v, w, t) V(u, v, w, t) d w d v d u - \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z k_{I,I}(u, v, w, T) \times \\
 & \times I^2(u, v, w, t) d w d v d u + \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z f_I(u, v, w) d w d v d u \quad (4a)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z V(u, v, w, t) d w d v d u = \frac{y z}{L_y L_z} \int_0^t \int_0^y \int_0^z D_V(x, v, w, T) \frac{\partial V(x, v, w, \tau)}{\partial x} d w d v d \tau + \frac{x z}{L_x L_z} \times \\
 & \times \int_0^t \int_0^x \int_0^z D_V(u, y, w, T) \frac{\partial V(u, y, w, \tau)}{\partial x} d w d u d \tau + \frac{x y}{L_x L_y} \int_0^t \int_0^x \int_0^y D_V(u, v, z, T) \frac{\partial V(u, v, z, \tau)}{\partial z} d v d u d \tau - \\
 & - \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z k_{I,V}(u, v, w, T) I(u, v, w, t) V(u, v, w, t) d w d v d u - \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z k_{V,V}(u, v, w, T) \times \\
 & \times V^2(u, v, w, t) d w d v d u + \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z f_V(u, v, w) d w d v d u.
 \end{aligned}$$

Now we consider solutions of the above integro-differential equations as the following series

$$\rho_0(x, y, z, t) = \sum_{n=1}^N a_{np} c_n(x) c_n(y) c_n(z) e_{np}(t).$$

Here a_{np} are coefficients, which should be determined. After substitution of the series into Eqs. (4a) one can obtain

$$\begin{aligned} \frac{xyz}{\pi^3} \sum_{n=1}^N \frac{a_{nl}}{n^3} s_n(x) s_n(y) s_n(z) e_{nl}(t) = & -\frac{yz\pi}{L_x L_y L_z} \sum_{n=1}^N a_{nl} \int_0^t e_{nl}(\tau) \int_{L_y}^y c_n(y) \int_{L_z}^z c_n(z) D_I(x, v, w, T) d w d v d \tau \times \\ & \times s_n(x) - \frac{xz\pi}{L_x L_y L_z} \sum_{n=1}^N a_{nl} s_n(y) \int_0^t e_{nl}(\tau) \int_{L_x}^x c_n(x) \int_{L_z}^z c_n(z) D_I(u, y, w, T) d w d u d \tau - \frac{xy\pi}{L_x L_y L_z} \sum_{n=1}^N a_{nl} s_n(z) \times \\ & \times \int_0^t e_{nl}(\tau) \int_{L_x}^x c_n(x) \int_{L_y}^y c_n(y) D_I(u, v, z, T) d v d u d \tau - \frac{xyz}{L_x L_y L_z} \int \int \int \left[\sum_{n=1}^N a_{nl} c_n(u) c_n(v) c_n(w) e_{nl}(t) \right]^2 \times \\ & \times k_{I,I}(u, v, v, T) d w d v d u - \frac{xyz}{L_x L_y L_z} \int \int \int \sum_{n=1}^N a_{nl} c_n(u) c_n(v) c_n(w) e_{nl}(t) \sum_{n=1}^N a_{nv} c_n(u) c_n(v) c_n(w) \times \\ & \times e_{nv}(t) k_{I,V}(u, v, v, T) d w d v d u + \frac{xyz}{L_x L_y L_z} \int \int \int f_I(u, v, w) d w d v d u \quad (12) \end{aligned}$$

$$\begin{aligned} \frac{xyz}{\pi^3} \sum_{n=1}^N \frac{a_{nv}}{n^3} s_n(x) s_n(y) s_n(z) e_{nv}(t) = & -\frac{yz\pi}{L_x L_y L_z} \sum_{n=1}^N a_{nv} \int_0^t e_{nv}(\tau) \int_{L_y}^y c_n(y) \int_{L_z}^z c_n(z) D_V(x, v, w, T) d w d v d \tau \times \\ & \times s_n(x) - \frac{xz\pi}{L_x L_y L_z} \sum_{n=1}^N a_{nv} s_n(y) \int_0^t e_{nv}(\tau) \int_{L_x}^x c_n(x) \int_{L_z}^z c_n(z) D_V(u, y, w, T) d w d u d \tau - \frac{xy\pi}{L_x L_y L_z} \sum_{n=1}^N a_{nv} s_n(z) \times \\ & \times \int_0^t e_{nv}(\tau) \int_{L_x}^x c_n(x) \int_{L_y}^y c_n(y) D_V(u, v, z, T) d v d u d \tau - \frac{xyz}{L_x L_y L_z} \int \int \int \left[\sum_{n=1}^N a_{nv} c_n(u) c_n(v) c_n(w) e_{nv}(t) \right]^2 \times \\ & \times k_{V,V}(u, v, v, T) d w d v d u - \frac{xyz}{L_x L_y L_z} \int \int \int \sum_{n=1}^N a_{nv} c_n(u) c_n(v) c_n(w) e_{nv}(t) \sum_{n=1}^N a_{nv} c_n(u) c_n(v) c_n(w) \times \\ & \times e_{nv}(t) k_{I,V}(u, v, v, T) d w d v d u + \frac{xyz}{L_x L_y L_z} \int \int \int f_V(u, v, w) d w d v d u. \end{aligned}$$

We use orthogonality condition of functions in the above series on scale of the heterostructure to calculate coefficients a_{np} . Using the condition gives a possibility to obtain equations for calculation the above coefficients for any quantity N of terms of considered series

$$\begin{aligned} -\frac{L_x^2 L_y^2 L_z^2}{\pi^5} \sum_{n=1}^N \frac{a_{nl}}{n^6} e_{nl}(t) = & -\frac{1}{2\pi L_x} \sum_{n=1}^N \frac{a_{nl}}{n^2} \int_0^t e_{nl}(\tau) \int_0^{L_x} [1 - c_n(2x)] \int_0^{L_y} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \times \\ & \times \int_0^{L_z} D_I(x, y, z, T) \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z) - 1] \right\} d z d y d x d \tau - \frac{1}{2\pi L_y} \sum_{n=1}^N \frac{a_{nl}}{n^2} \int_0^t \int_0^{L_x} \left\{ \frac{L_x}{\pi n} [c_n(2x) - 1] + L_x + \right. \\ & \left. + x s_n(2x) \right\} \int_0^{L_y} \int_0^{L_z} D_I(x, y, z, T) \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} d z [1 - c_n(2y)] d y d x e_{nl}(\tau) d \tau - \\ & - \frac{1}{2\pi L_z} \sum_{n=1}^N \frac{a_{nl}}{n^2} \int_0^t e_{nl}(\tau) \int_0^{L_x} \left\{ L_x + x s_n(2x) + \frac{L_x}{2\pi n} [c_n(2x) - 1] \right\} \int_0^{L_y} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \times \end{aligned}$$

$$\begin{aligned}
 & \times \int_0^{L_z} [1 - c_n(2z)] D_I(x, y, z, T) dz dy dx d\tau - \sum_{n=1}^N a_{nl}^2 e_{nl}(2t) \int_0^{L_x} \left\{ L_x + \frac{L_x}{2\pi n} [c_n(2x) - 1] + x s_n(2x) \right\} \times \\
 & \times \int_0^{L_y} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \int_0^{L_z} k_{I,I}(x, y, z, T) \left\{ L_z + \frac{L_z}{2\pi n} [c_n(2z) - 1] + z s_n(2z) \right\} dz dy dx - \\
 & - \sum_{n=1}^N a_{nl} a_{nv} e_{nl}(t) e_{nv}(t) \int_0^{L_x} \left\{ L_x + x s_n(2x) + \frac{L_x}{2\pi n} [c_n(2x) - 1] \right\} \int_0^{L_y} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \times \\
 & \times \int_0^{L_z} k_{I,V}(x, y, z, T) \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} dz dy dx + \sum_{n=1}^N \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \times \\
 & \times \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} f_I(x, y, z, T) \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} dz dy dx \quad (13) \\
 & - \frac{L_x^2 L_y^2 L_z^2}{\pi^5} \sum_{n=1}^N \frac{a_{nv}}{n^6} e_{nv}(t) = - \frac{1}{2\pi L_x} \sum_{n=1}^N \frac{a_{nv}}{n^2} \int_0^{L_x} e_{nv}(\tau) \int_0^{L_y} [1 - c_n(2x)] \int_0^{L_z} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \times \\
 & \times \int_0^{L_z} D_V(x, y, z, T) \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z) - 1] \right\} dz dy dx d\tau - \frac{1}{2\pi L_y} \sum_{n=1}^N \frac{a_{nv}}{n^2} \int_0^{L_x} \int_0^{L_z} \left\{ \frac{L_x}{\pi n} [c_n(2x) - 1] + L_x + \right. \\
 & \left. + x s_n(2x) \right\} \int_0^{L_z} D_V(x, y, z, T) \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} dz [1 - c_n(2y)] dy dx e_{nv}(\tau) d\tau - \\
 & - \frac{1}{2\pi L_z} \sum_{n=1}^N \frac{a_{nv}}{n^2} \int_0^{L_x} e_{nv}(\tau) \int_0^{L_y} \left\{ L_x + x s_n(2x) + \frac{L_x}{2\pi n} [c_n(2x) - 1] \right\} \int_0^{L_z} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \times \\
 & \times \int_0^{L_z} [1 - c_n(2z)] D_V(x, y, z, T) dz dy dx d\tau - \sum_{n=1}^N a_{nv}^2 e_{nv}(2t) \int_0^{L_x} \left\{ L_x + \frac{L_x}{2\pi n} [c_n(2x) - 1] + x s_n(2x) \right\} \times \\
 & \times \int_0^{L_y} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \int_0^{L_z} k_{V,V}(x, y, z, T) \left\{ L_z + \frac{L_z}{2\pi n} [c_n(2z) - 1] + z s_n(2z) \right\} dz dy dx - \\
 & - \sum_{n=1}^N a_{nl} a_{nv} e_{nl}(t) e_{nv}(t) \int_0^{L_x} \left\{ L_x + x s_n(2x) + \frac{L_x}{2\pi n} [c_n(2x) - 1] \right\} \int_0^{L_y} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \times \\
 & \times \int_0^{L_z} k_{I,V}(x, y, z, T) \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} dz dy dx + \sum_{n=1}^N \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \times \\
 & \times \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} f_V(x, y, z, T) \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} dz dy dx.
 \end{aligned}$$

Final relations for the above parameters takes the form

$$a_{nl} = -\frac{b_3 + A}{4b_4} \pm \sqrt{\frac{(b_3 + A)^2}{4} - 4b_4 \left(y + \frac{b_3 y - \gamma_{nv} \lambda_{nl}^2}{A} \right)}, \quad a_{nv} = -\frac{\gamma_{nl} a_{nl}^2 + \delta_{nl} a_{nl} + \lambda_{nl}}{\chi_{nl} a_{nl}}. \quad (14)$$

Here $\gamma_{np} = e_{np}(2t) \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} k_{p,p}(x, y, z, T) \left\{ L_x + x s_n(2x) + \frac{L_x}{2\pi n} [c_n(2x) - 1] \right\} \left\{ y s_n(2y) + L_y + \frac{L_y}{2\pi n} \times \right.$

$$\begin{aligned}
 & \times [c_n(2y)-1] \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z)-1] \right\} d z d y d x, \delta_{np} = \frac{1}{2\pi L_x n^2} \int_0^t e_{np}(\tau) \int_0^{L_x} \int_0^{L_y} \{ y s_n(y) + \\
 & + \frac{L_y}{2\pi n} [c_n(y)-1] \} \int_0^{L_z} \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z)-1] \right\} D_\rho(x, y, z, T) d z d y [1 - c_n(2x)] d x d \tau + \frac{1}{2\pi L_y} \times \\
 & \times \frac{1}{2n^2} \int_0^t e_{np}(\tau) \int_0^{L_x} \left\{ L_x + x s_n(2x) + \frac{L_x}{\pi n} [c_n(2x)-1] \right\} \int_0^{L_z} [1 - c_n(2y)] \int_0^{L_y} D_\rho(x, y, z, T) \left\{ \frac{L_z}{2\pi n} [c_n(2z)-1] + \right. \\
 & + z s_n(2z) + L_z \} d z d y d x d \tau + \frac{1}{2\pi L_z n^2} \int_0^t e_{np}(\tau) \int_0^{L_x} \left\{ x s_n(2x) + L_x + \frac{L_x}{\pi n} [c_n(2x)-1] \right\} \int_0^{L_y} \{ y s_n(y) + \\
 & + L_y + \frac{L_y}{2\pi n} [c_n(y)-1] \} \int_0^{L_z} [1 - c_n(2z)] D_\rho(x, y, z, T) d z d y d x d \tau - \frac{L_x^2 L_y^2 L_z^2}{\pi^5 n^6} e_{np}(t), \chi_{nIV} = e_{nl}(t) \times \\
 & \times e_{nV}(t) \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x)-1] \right\} \int_0^{L_y} \left\{ L_y + \frac{L_y}{2\pi n} [c_n(2y)-1] + y s_n(2y) \right\} \int_0^{L_z} k_{I,V}(x, y, z, T) \{ L_z + z s_n(2z) + \\
 & + \frac{L_z}{2\pi n} [c_n(2z)-1] \} d z d y d x, \lambda_{np} = \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x)-1] \right\} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y)-1] \right\} \int_0^{L_z} \{ z \times \\
 & \times s_n(z) + \frac{L_z}{\pi n} [c_n(z)-1] \} f_\rho(x, y, z, T) d z d y d x, b_4 = \gamma_{nV} \gamma_{nI}^2 - \gamma_{nI} \chi_{nI}^2, b_3 = 2\gamma_{nV} \gamma_{nI} \delta_{nI} - \delta_{nI} \chi_{nI}^2 - \\
 & - \delta_{nV} \chi_{nI} \gamma_{nI}, b_2 = \gamma_{nV} \delta_{nI}^2 + 2\lambda_{nI} \gamma_{nV} \gamma_{nI} - \delta_{nV} \chi_{nI} \delta_{nI} + (\lambda_{nV} - \lambda_{nI}) \chi_{nI}^2, b_1 = 2\lambda_{nI} \gamma_{nV} \delta_{nI} - \delta_{nV} \chi_{nI} \lambda_{nI}, \\
 & A = \sqrt{8y + b_3^2 - 4b_2}, p = \frac{3b_2 b_4 - b_3^2}{9b_4^2}, q = \frac{2b_3^3 - 9b_2 b_3 + 27b_1 b_4^2}{54b_4^3}, y = \sqrt[3]{\sqrt{q^2 + p^3} - q} - \sqrt[3]{\sqrt{q^2 + p^3} + q} - \\
 & - b_3/3b_4.
 \end{aligned}$$

Now we will calculate distributions of concentrations of simplest complexes of radiation defects as the following functional series

$$\Phi_{\rho 0}(x, y, z, t) = \sum_{n=1}^N a_{n\Phi\rho} c_n(x) c_n(y) c_n(z) e_{np}(t) \tag{15}$$

with not yet known coefficients $a_{n\Phi\rho}$. To calculate these coefficient we transform the Eqs.(6) to the following integro-differential form

$$\begin{aligned}
 & \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z \Phi_I(u, v, w, t) d w d v d u = \frac{y z}{L_y L_z} \int_0^t \int_0^{L_x} \int_0^{L_z} D_{\Phi I}(x, v, w, T) \frac{\partial \Phi_I(x, v, w, \tau)}{\partial x} d w d v d \tau + \\
 & + \frac{x z}{L_x L_z} \int_0^t \int_0^{L_x} \int_0^{L_z} D_{\Phi I}(u, y, w, T) \frac{\partial \Phi_I(u, y, w, \tau)}{\partial y} d w d u d \tau + \int_0^t \int_0^{L_x} \int_0^{L_y} D_{\Phi I}(u, v, z, T) \frac{\partial \Phi_I(u, v, z, \tau)}{\partial z} d v d u d \tau \times \\
 & \times \frac{x y}{L_x L_y} + \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z k_{I,I}(u, v, w, T) I^2(u, v, w, \tau) d w d v d u - \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z k_I(u, v, w, T) \times \\
 & \times I(u, v, w, \tau) d w d v d u + \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z f_{\Phi I}(u, v, w) d w d v d u \tag{6a}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z \Phi_V(u, v, w, t) d w d v d u = \frac{y z}{L_y L_z} \int_0^t \int_0^{L_x} \int_0^{L_z} D_{\Phi V}(x, v, w, T) \frac{\partial \Phi_V(x, v, w, \tau)}{\partial x} d w d v d \tau + \\
 & + \frac{x z}{L_x L_z} \int_0^t \int_0^{L_x} \int_0^{L_z} D_{\Phi V}(u, y, w, T) \frac{\partial \Phi_V(u, y, w, \tau)}{\partial y} d w d u d \tau + \int_0^t \int_0^{L_x} \int_0^{L_y} D_{\Phi V}(u, v, z, T) \frac{\partial \Phi_V(u, v, z, \tau)}{\partial z} d v d u d \tau \times
 \end{aligned}$$

$$\begin{aligned} & \times \frac{x y}{L_x L_y} + \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z k_{V,V}(u, v, w, T) V^2(u, v, w, \tau) d w d v d u - \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z k_V(u, v, w, T) \times \\ & \quad \times V(u, v, w, \tau) d w d v d u + \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z f_{\Phi V}(u, v, w) d w d v d u . \end{aligned}$$

Further we substitute the previously considered series in the Eqs.(6a). In this situation we obtain the following equation

$$\begin{aligned} & -x y z \sum_{n=1}^N \frac{a_{n\Phi I}}{\pi^3 n^3} s_n(x) s_n(y) s_n(z) e_{nI}(t) = -\frac{y z \pi}{L_x L_y L_z} \sum_{n=1}^N n a_{n\Phi I} s_n(x) e_{nI}(t) \int_0^t \int_0^y c_n(v) \int_0^z D_{\Phi I}(x, v, w, T) \times \\ & \quad \times c_n(w) d w d v d \tau - \frac{x z \pi}{L_x L_y L_z} \sum_{n=1}^N a_{n\Phi I} n s_n(y) e_{n\Phi I}(t) \int_0^t \int_0^x \int_0^z c_n(u) c_n(w) D_{\Phi I}(u, v, w, T) d w d u d \tau - \\ & \quad - \frac{x y \pi}{L_x L_y L_z} \sum_{n=1}^N n a_{n\Phi I} s_n(z) e_{n\Phi I}(t) \int_0^t \int_0^x \int_0^y c_n(u) c_n(v) D_{\Phi I}(u, v, z, T) d v d u d \tau + \int_0^x \int_0^y \int_0^z k_{I,I}(u, v, w, T) \times \\ & \quad \times I^2(u, v, w, \tau) d w d v d u - \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z k_I(u, v, w, T) I(u, v, w, \tau) d w d v d u + \frac{x y z}{L_x L_y L_z} \times \\ & \quad \times \int_0^x \int_0^y \int_0^z f_{\Phi I}(u, v, w) d w d v d u \end{aligned} \quad (16)$$

$$\begin{aligned} & -x y z \sum_{n=1}^N \frac{a_{n\Phi V}}{\pi^3 n^3} s_n(x) s_n(y) s_n(z) e_{nV}(t) = -\frac{y z \pi}{L_x L_y L_z} \sum_{n=1}^N n a_{n\Phi V} s_n(x) e_{nV}(t) \int_0^t \int_0^y c_n(v) \int_0^z D_{\Phi V}(x, v, w, T) \times \\ & \quad \times c_n(w) d w d v d \tau - \frac{x z \pi}{L_x L_y L_z} \sum_{n=1}^N a_{n\Phi V} n s_n(y) e_{n\Phi V}(t) \int_0^t \int_0^x \int_0^z c_n(u) c_n(w) D_{\Phi V}(u, v, w, T) d w d u d \tau - \\ & \quad - \frac{x y \pi}{L_x L_y L_z} \sum_{n=1}^N n a_{n\Phi V} s_n(z) e_{n\Phi V}(t) \int_0^t \int_0^x \int_0^y c_n(u) c_n(v) D_{\Phi V}(u, v, z, T) d v d u d \tau + \int_0^x \int_0^y \int_0^z k_{V,V}(u, v, w, T) \times \\ & \quad \times V^2(u, v, w, \tau) d w d v d u - \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z k_V(u, v, w, T) V(u, v, w, \tau) d w d v d u + \frac{x y z}{L_x L_y L_z} \times \\ & \quad \times \int_0^x \int_0^y \int_0^z f_{\Phi V}(u, v, w) d w d v d u . \end{aligned}$$

We use orthogonality condition of functions in the above series on scale of the heterostructure to calculate coefficients $a_{n\Phi}$. Using the condition gives a possibility to obtain equations for calculation the above coefficients for any quantity N of terms of considered series

$$\begin{aligned} & -\frac{L_x^2 L_y^2 L_z^2}{\pi^5} \sum_{n=1}^N \frac{a_{n\Phi I}}{n^6} e_{n\Phi I}(t) = -\frac{1}{2\pi L_x} \sum_{n=1}^N \frac{a_{n\Phi I}}{n^2} \int_0^t \int_0^{L_x} [1 - c_n(2x)] \int_0^{L_y} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \times \\ & \quad \times \int_0^{L_z} D_{\Phi I}(x, y, z, T) \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z) - 1] \right\} d z d y d x e_{n\Phi I}(\tau) d \tau - \frac{1}{2\pi} \sum_{n=1}^N \frac{a_{n\Phi I}}{n^2 L_y} \int_0^t \int_0^{L_y} \left\{ x s_n(2x) + L_x + \right. \\ & \quad \left. + L_x \frac{c_n(2x) - 1}{2\pi n} \right\} \int_0^{L_z} [1 - c_n(2y)] \int_0^{L_z} D_{\Phi I}(x, y, z, T) \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z) - 1] \right\} d z d y d x e_{n\Phi I}(\tau) d \tau - \frac{1}{\pi L_x} \times \end{aligned}$$

$$\begin{aligned}
 & \times \sum_{n=1}^N \frac{a_{n\Phi I}}{2n^2} \int_0^t e_{n\Phi I}(\tau) \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{2\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] + L_y \right\} \int_0^{L_z} D_{\Phi I}(x, y, z, T) \times \\
 & \times [1 - c_n(2y)] d z d y d x d \tau + \sum_{n=1}^N \int_0^t e_{n\Phi I}(\tau) \int_0^{L_x} \left\{ \frac{L_x}{2\pi n} [c_n(x) - 1] + x s_n(x) \right\} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{2\pi n} [c_n(y) - 1] \right\} \times \\
 & \times \frac{a_{n\Phi I}}{n^3 \pi^3} \int_0^{L_z} I^2(x, y, z, t) k_{I,I}(x, y, z, T) \left\{ \frac{L_z}{2\pi n} [c_n(z) - 1] + z s_n(z) \right\} d z d y d x - \sum_{n=1}^N \int_0^t e_{n\Phi I}(\tau) \int_0^{L_x} \left\{ x s_n(x) + \right. \\
 & + \frac{L_x}{2\pi n} [c_n(x) - 1] \left. \right\} \frac{a_{n\Phi I}}{n^3 \pi^3} \int_0^{L_y} \left\{ \frac{L_y}{2\pi n} [c_n(y) - 1] + y s_n(y) \right\} \int_0^{L_z} k_I(x, y, z, T) I(x, y, z, t) \left\{ \frac{L_z}{2\pi n} [c_n(z) - 1] + \right. \\
 & + z s_n(z) \left. \right\} d z d y d x + \sum_{n=1}^N \frac{a_{n\Phi I}}{\pi^3 n^3} \int_0^t e_{n\Phi I}(\tau) \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{2\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{2\pi n} [c_n(y) - 1] \right\} \times \\
 & \times \int_0^{L_z} \left\{ \frac{L_z}{2\pi n} [c_n(z) - 1] + z s_n(z) \right\} f_{\Phi I}(x, y, z) d z d y d x \tag{17}
 \end{aligned}$$

$$\begin{aligned}
 & - \frac{L_x L_y L_z}{\pi^5} \sum_{n=1}^N \frac{a_{n\Phi V}}{n^6} e_{n\Phi V}(t) = - \frac{1}{2\pi L_x} \sum_{n=1}^N \frac{a_{n\Phi V}}{n^2} \int_0^t \int_0^{L_x} [1 - c_n(2x)] \int_0^{L_y} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \times \\
 & \times \int_0^{L_z} D_{\Phi V}(x, y, z, T) \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z) - 1] \right\} d z d y d x e_{n\Phi V}(\tau) d \tau - \frac{1}{2\pi} \sum_{n=1}^N \frac{a_{n\Phi V}}{n^2 L_y} \int_0^t \int_0^{L_x} \left\{ x s_n(2x) + L_x + \right. \\
 & + L_x \frac{c_n(2x) - 1}{2\pi n} \left. \right\} \int_0^{L_y} [1 - c_n(2y)] \int_0^{L_z} D_{\Phi V}(x, y, z, T) \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z) - 1] \right\} d z d y d x e_{n\Phi V}(\tau) d \tau - \frac{1}{\pi L_x} \times \\
 & \times \sum_{n=1}^N \frac{a_{n\Phi V}}{2n^2} \int_0^t e_{n\Phi V}(\tau) \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{2\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] + L_y \right\} \int_0^{L_z} D_{\Phi V}(x, y, z, T) \times \\
 & \times [1 - c_n(2y)] d z d y d x d \tau + \sum_{n=1}^N \int_0^t e_{n\Phi V}(\tau) \int_0^{L_x} \left\{ \frac{L_x}{2\pi n} [c_n(x) - 1] + x s_n(x) \right\} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{2\pi n} [c_n(y) - 1] \right\} \times \\
 & \times \frac{a_{n\Phi V}}{n^3 \pi^3} \int_0^{L_z} V^2(x, y, z, t) k_{V,V}(x, y, z, T) \left\{ \frac{L_z}{2\pi n} [c_n(z) - 1] + z s_n(z) \right\} d z d y d x - \sum_{n=1}^N \int_0^t e_{n\Phi V}(\tau) \int_0^{L_x} \left\{ x s_n(x) + \right. \\
 & + \frac{L_x}{2\pi n} [c_n(x) - 1] \left. \right\} \frac{a_{n\Phi V}}{n^3 \pi^3} \int_0^{L_y} \left\{ \frac{L_y}{2\pi n} [c_n(y) - 1] + y s_n(y) \right\} \int_0^{L_z} k_V(x, y, z, T) V(x, y, z, t) \left\{ \frac{L_z}{2\pi n} [c_n(z) - 1] + \right. \\
 & + z s_n(z) \left. \right\} d z d y d x + \sum_{n=1}^N \frac{a_{n\Phi V}}{\pi^3 n^3} \int_0^t e_{n\Phi V}(\tau) \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{2\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{2\pi n} [c_n(y) - 1] \right\} \times \\
 & \times \int_0^{L_z} \left\{ \frac{L_z}{2\pi n} [c_n(z) - 1] + z s_n(z) \right\} f_{\Phi V}(x, y, z) d z d y d x .
 \end{aligned}$$

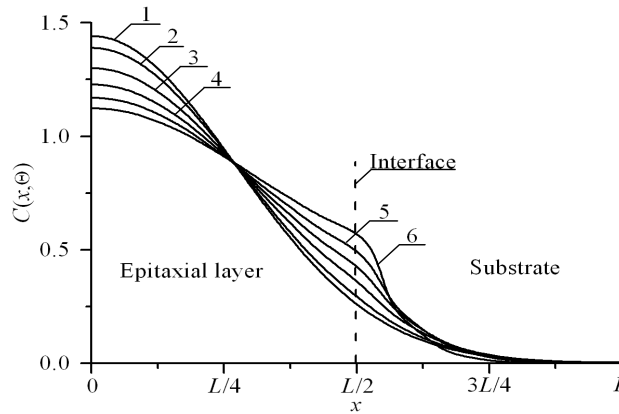


Fig.2a. Spatial distributions of concentration of dopant after infusion and annealing with the same annealing time before and after interface between layers of heterostructure. Curve 1 is the distribution of concentration of dopant in homogenous sample with averaged dopant diffusion coefficient D_0 . Curves 2-6 are the distribution of concentration of dopant in heterostructure with increasing difference between values of diffusion coefficient. Value of dopant diffusion coefficient in the substrate is smaller, than in epitaxial the layer

3. DISCUSSION

In this section analysis of distributions of concentration of dopant, infused (see Fig. 2a) or implanted (see Fig. 2b) into epitaxial layer have been done. Annealing time is the same for all curves of these figures. One can find from these figures, that absolute value of gradient of concentration of dopant increases due to presents an interface between layers of heterostructure. In this situation dimensions of the considered transistors decreases. At the same time homogeneity of concentration of dopant in enriched area increases.

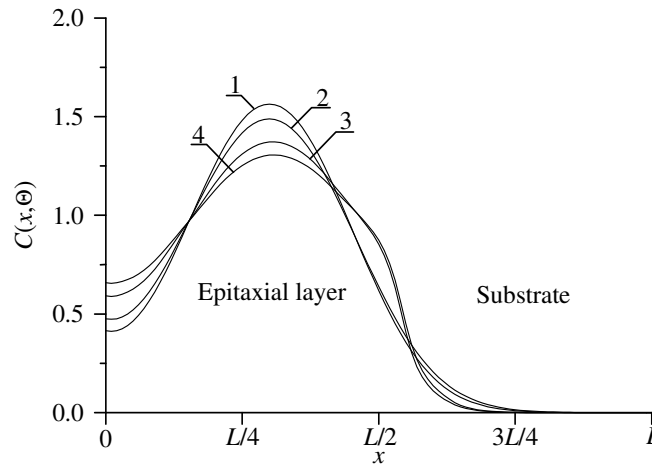


Fig.2b. Spatial distributions of concentration of dopant after implantation and annealing with the same annealing time before and after interface between layers of heterostructure. Annealing time for curves 1 and 3 is equal to $\Theta = 0.0048(L_x^2 + L_y^2 + L_z^2)/D_0$. Annealing time for curves 2 and 4 is equal to $\Theta = 0.0057(L_x^2 + L_y^2 + L_z^2)/D_0$. Curves 1 and 2 are the distributions of concentration of dopant in homogenous sample. Curves 3 and 4 are distributions of concentration of dopant in heterostructure under condition, when value of dopant diffusion coefficient in the substrate is smaller, than in epitaxial the layer

To estimate optimal annealing time we estimate decreasing of absolute value of gradient of concentration of dopant near interface between layers of the heterostructure with increasing of annealing time. Decreasing of annealing time leads to increasing inhomogeneity of distribution of concentration of dopant. Estimation of the compromise value of annealing time has been done by using recently introduced criterion [14,15]. To use the criterion we approximate real spatial distribution of concentration of dopant by idealized step-wise function $\psi(x,y,z)$. After that we estimate the required optimal annealing time by minimization of mean-square error [16-20]

$$U = \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} [C(x, y, z, \Theta) - \psi(x, y, z)]^2 dz dy dx. \quad (18)$$

Minimization of the above mean-squared error leads to dependences of optimal annealing time on parameters, which are presented on Figs. 3. It should be noted, that radiation defects, generated during ion implantation, should be annealed. After ideal optimization of annealing time the implanted dopant should achieve the interface between layers of heterostructure. If annealing time is smaller, it is attracted an interest to make additional annealing to achieve the interface. The Fig. 3b shows dependences of additional annealing time.

The figures shows, that optimal annealing time of implanted dopant is smaller in comparison with optimal annealing time of infused dopant. If the considered heterostructure have been doped by diffusion, any radiation damage of materials of layers is absent. On the other hand radiation processing of materials of heterostructure (including of ion implantation) leads to decreasing of mismatch-induced stress in the processed heterostructure [20].

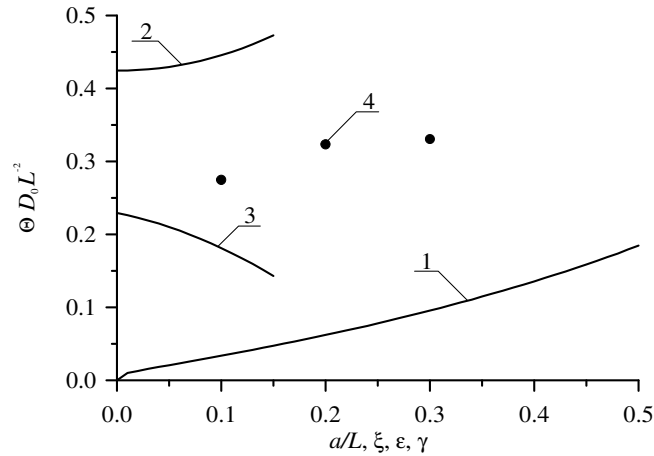


Fig. 3a. Dependences of optimal annealing time of infused dopant on several parameters. Dependence of the optimal annealing time on normalized thickness of the epitaxial layer a/L describes by curve 1 for $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in both parts of heterostructure. Curve 2 describes dependence of the optimal annealing time on the parameter ϵ for $a/L = 1/2$ and $\xi = \gamma = 0$. Curve 3 describes dependence of the optimal annealing time on the parameter ξ for $a/L = 1/2$ and $\epsilon = \gamma = 0$. Curve 4 describes dependence of the optimal annealing time on parameter γ for $a/L = 1/2$ and $\epsilon = \xi = 0$

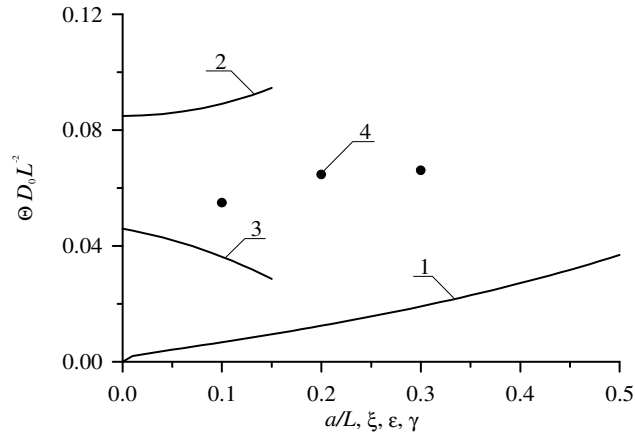


Fig.3b. Dependences of optimal annealing time of implanted dopant on several parameters. Dependence of the optimal annealing time on normalized thickness of the epitaxial layer a/L describes by curve 1 for $\xi=\gamma=0$ for equal to each other values of dopant diffusion coefficient in both parts of heterostructure. Curve 2 describes dependence of the optimal annealing time on the parameter ε for $a/L=1/2$ and $\xi=\gamma=0$. Curve 3 describes dependence of the optimal annealing time on the parameter ξ for $a/L=1/2$ and $\varepsilon=\gamma=0$. Curve 4 describes dependence of the optimal annealing time on parameter γ for $a/L=1/2$ and $\varepsilon=\xi=0$

4. CONCLUSIONS

In this paper we formulate several recommendations to optimize manufacture heterotransistor with several source based on prognosis of time varying of spatial distributions of concentrations of infused and implanted dopants in specific heterostructure. We also introduce analytical approach to prognosis diffusion and ion types of doping with account variation in space and time parameters of technological parameters and nonlinearity of mass and heat transport.

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