A FUZZY INVENTORY MODEL WITH LOT SIZE DEPENDENT ORDERING COST IN HEALTHCARE INDUSTRIES

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ABSTRACT

The classical Harris - Wilson inventory model assumes that all the cost associated with the model was taken to be constant and which does not dependent on any quantity ordered. In this paper we have taken Ordering cost, holding cost and order quantity all are triangular fuzzy numbers. Graded mean integration representation method is used for defuzzification. In this paper, we consider an inventory model where the ordering cost depends on the size of the lot and increases in steps as the lot size increases. The main goal of this research is to reduce the healthcare cost and without sacrificing customer service. An algorithm is developed to find the economic order quantity along with numerical examples in pharmaceutical company.

KEYWORDS:

Inventory, Lot size, Ordering Cost, Fuzzy inventory model, Triangular fuzzy numbers, Defuzzification.

Subject Classification Codes: 90B05

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1. INTRODUCTION

Kanti Swarup et al. [13] discussed about Inventory model, such as the basic of inventories. The total of an objects, a pharmaceutical company has in store at a particular time is known as inventory or in terms of money it can be defined as the total investment over all the resources stocked in the pharmaceutical company at any particular time. Inventory possibly in the type of, unprocessed material inventory, in process inventory, fulfilled goods inventory, etc.

As a lot of money is occupied in the inventories along with their high carrying costs, the pharmaceutical companies cannot afford to have any money tied in excess inventories. Any excessive investment in pharmaceutical inventories may prove to be a serious drag on the successful working of a healthcare organization. Therefore there is a must to deal with pharmaceutical inventories more successfully to free the excessive amount of capital engaged in the pharmaceutical materials.
Inventory control is the supervision of supply, storage and accessibility of items in order to ensure an adequate supply without excessive oversupply. It can also be referred as internal control of an accounting procedure or system considered to promote good organization or assure the achievement of a policy or safeguards assets or avoid fraud and error, etc. In financial area, the inventory control problem, which plan to reduce overhead cost without hurting sales. In the field of loss anticipation, systems designed to introduce technical barriers to shoplifting. Also it answers the following basic questions of any supply chain: (1) what to order? (2) When to order? (3) Where to order? (4) How much to hold in stock? so as to increase finances.

In order to control inventories properly, one has to consider all cost elements that are linked with the inventories. There are few such cost essentials, which do affect cost of inventory. The total cost of holding inventory is called Carrying Inventory. This contain warehousing costs such as hire fee, utilities and salaries, financial costs such as opportunity cost, and inventory costs related to perishability, pilferage, reduction and assurance. Ordering cost is cost of ordering raw materials for pharmaceutical production purposes. These include cost of placing a purchase order, costs of check up of received batches, certification costs, etc.

**Economic order quantity (EOQ)** is the size of order which minimizes the total annual costs of carrying inventory and cost of ordering. Stock out cost is costs incurred when an item is out of stock. These costs include the lost contribution boundary on sales plus lost customer. The lead time is the time gap among placing of an order and its actual arrival in the pharmaceutical inventory. Demand is the number of units required per period. Demand refers to how much (quantity) of a manufactured goods or service is preferred by buyers at various prices.

One of the challenging problems for Healthcare researchers and practitioners is to study and analyze the inventory systems. Those systems cannot only decrease the costs, but also decrease stock outs and develop patient satisfactions. Medicine deficiencies and inappropriate use of pharmaceuticals can not only lead to economic losses but also have a considerable impact on patients. A lot of healthcare systems and hospitals familiarity difficulties in achieving these targets as they have not attend to how medicines are handled, supplied and used to save lives and develop health.

### 2. LITERATURE REVIEW

Inventory consists of usable but idle resources. There are many real life applications where this situation takes place. Let us consider an example that the freight costs which is required to be paid by the buyer. Usually the pharmaceuticals are shipped in a truck, train, van etc and when pharmaceuticals are received, they are checked, inspected because of the pharmaceuticals may be expired. If we use the expired one, we suffer lot. Therefore the ordering cost would in fact increase as the lot size increases.

The Harris – Wilson Formula [12] for determining the optimum lot size is the original and new model in the inventory systems. The Basic EOQ model considers three types of costs like cost of the product, ordering or set up cost and holding cost or carrying cost. A number of examiners deal with the modifications of these cost formations. It assumes that the ordering cost is constant and does not depend on the size of the lot. Vandana and B.K. Sharma [25], urbanized an inventory model for vendors partial permissible delay-in-payment connected to order quantity with deficiency, which is partial backlogged.

Gupta [11] thinks an inventory model wherever the ordering cost depends on the amount of the lot and rises in steps as the lot size rises. Gordon [10], Emery [8], Arrow [1] have given so many worth results in this field. A number of real locations carrying cost depends on the lot-size and different expansions of non-constant carrying cost can be seen in Beranek [2]. Karabi Dutta
Choudhury et al [14] discussed about an inventory model with lot size dependent or carrying cost. Parvathi and Gajalakshmi [17] discussed a fuzzy inventory model with lot size dependent carrying or holding cost. The detailed study of basic concepts about fuzzy sets and fuzzy logic derived by George J. Klir/Bo Yuan [9].

Vandana and B.K. Sharma [23] discussed an EPQ inventory model for non-instantaneous deteriorating items under trade credit strategy also Vandana and B.K. Sharma [24] developed an inventory model for Non-Instantaneous deteriorating items with quadratic demand time and deficiencies under trade credit strategy.

Pharmaceuticals represent a considerable part of health care costs, account for around 10% of annual health care expenditure in the USA and about $600 billion globally in 2009 [21]. R. Uthayakumar and S. Priyan [22], discussed for determining mainly positive solutions for inventory lot size, lead time, and the number of deliveries to achieve hospital Customer Service Level aims with the smallest total cost for the supply chain. J. K. Syed and L. A. Aziz [20] instead of the ordering cost and holding cost by fuzzy triangular number, the optimal order quantity is analyzed using Signed Distance Method for defuzzification.

Chien-Chang Chou [5] proposes a fuzzy backorder inventory model for solving the optimal order quantity inventory problem. Costs and quantities are uttered in trapezoidal fuzzy numbers. The Function Principle to utilize arithmetical process, the Graded Mean Integration Representation system to defuzzify, and the Kuhn-Tucker situations to find the best possible backorder amount and shortage amount for the fuzzy backorder amount inventory model.


Fuzzy sets in place of linguistic concepts such as low, medium, and high, are employed to define states of a variable. The membership function of a fuzzy set takes a quantity significance and may be sighted as a fuzzy number supplied they satisfy firm conditions. Fuzzy numbers are mostly applied on data analysis, artificial intelligence, and decision making. In particular, triangular fuzzy numbers are commonly used in applications and it is also easy to handle the difficulty. Thus the purpose of fuzzy set concepts on EOQ inventory models have been projected by many authors (e.g. [3, 4, 7, 16, 18, 26, and 27]).

The remainder of the paper is organized as follows. Part 3 provides the notations and assumptions used. Source of the Operation Research model is described in part 4 and the solution process is obtainable in part 5. An efficient algorithm is urbanized to find the optimal solution in part 6. A numerical example is provided in Section 7. Finally, we draw some conclusion in section 8.

### 3. NOTATIONS AND ASSUMPTIONS

We build up an Operation Research model using the notations and assumptions scheduled below. Further notations and assumptions are provided when necessary.

#### 3.1. Notations

- \( D \) – Total / Annual Demand
- \( S \) – Setup Cost / Ordering Cost (per order)
- \( H \) – Holding Cost / Carrying Cost for a unit for one year.
3.2 Assumptions

- Demand is known and constant.
- Shortages are not allowed.
- The order quantity is received instantaneously.
- Lead time is zero.
- Ordering/Setup cost, holding/carrying cost and order quantity all are triangular fuzzy numbers

![Figure 1. Graph of EOQ](image)

4. MATHEMATICAL MODEL FORMULATION

The Harris Wilson technique for deciding the best possible lot size, the amount in which an article of inventory should be procured or constructs is

\[ Q = \sqrt{\frac{2DS}{H}} \]  

Our endeavor is to introduce additional realistic inventory model for which an algorithm is developed. Our planned model is illustrated through numerical examples.

This system follows that,

1. The best possible amount reduces the sum of the annual setup/ordering cost and holding/carrying cost (figure 1) and
2. The total cost is
\[ TC(Q) = \frac{Q^*H}{2} + \frac{D*S}{Q} \]

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i.e. the costs considered are understood to be unchanging. This body of research thinks that the factors concerned in the EOQ model, such as the demand and the purchasing cost are crisp values or random variables. However, in certainty, the demand and the cost of the pharmaceutical things often change slightly from one cycle to another. Also, it is very tough to estimate the probability distribution of these variables due to a need of historical data. Instead, the cost parameters are frequently estimated based on experience and individual healthcare management decision. Thus, the fuzzy set theory, rather than the conventional probability theory, is fine matched to the inventory.

The paper developed by Gupta considered the demand and holding costs as constant and ordering cost is non constant in their model. To generate it more realistic we have taken ordering/setup cost, holding/carrying cost and order quantity as triangular fuzzy numbers. Graded mean integration representation technique is used for defuzzification.

Methodology

Fuzzy Numbers

A fuzzy subset of the real line \( R \), whose membership function \( M_f \) satisfies the following situation, is a generalized fuzzy number \( \tilde{A} \).

(i) \( M_f \) is a continuous mapping from \( R \) to the closed interval \([0, 1]\),
(ii) \( M_f = 0, \quad -\infty < x \leq a_1 \)
(iii) \( M_f = L(x) \) is strictly increasing on \([a_1, a_2]\)
(iv) \( M_f = W_A, \quad a_2 \leq x \leq a_3 \)
(v) \( M_f = R(x) \) is strictly decreasing on \([a_3, a_4]\)
(vi) \( M_f = 0, \quad a_4 \leq x \leq \infty \)

Where \( 0 < W_A \leq 1 \) and \( a_1, a_2, a_3 \) and \( a_4 \) are real numbers. Moreover, this kind of generalized fuzzy number is denoted as \( \tilde{A}=(a_1, a_2, a_3, a_4; W_A)_{LR} \); when \( W_A = 1 \), it can be simplified as \( \tilde{A}=(a_1, a_2, a_3, a_4; W_A)_{LR} \).

The Triangular fuzzy number

The fuzzy set \( \tilde{A}=(a_1, a_2, a_3) \) where \( a_1 < a_2 < a_3 \) and described on \( R \), is called the triangular fuzzy number, the membership function of \( \tilde{A} \) is given by
The function principle was introduced by Shan-Huo Chen [19] to treat fuzzy arithmetical operations. This principle is used for the process for addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{A}=(a_1,a_2,a_3)$ and $\tilde{B}=(b_1,b_2,b_3)$ are two triangular fuzzy numbers. Then

(i) the addition of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

where $a_1$, $a_2$, $a_3$, $b_1$, $b_2$, $b_3$ are any real numbers.

(ii) The multiplication of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \times \tilde{B} = (a_1b_1, a_2b_2, a_3b_3)$$

where $a_1$, $a_2$, $a_3$, $b_1$, $b_2$, $b_3$ are all non zero positive real numbers.

(iii) $-\tilde{B} = (-b_3, -b_2, -b_1)$, the subtraction of $\tilde{B}$ from $\tilde{A}$ is

$$\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$$

where $a_1$, $a_2$, $a_3$, $b_1$, $b_2$, $b_3$ are any real numbers.
(iv) \( \frac{1}{B} = \tilde{B}^{-1} = (\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}) \) where \( b_1, b_2, b_3 \) are all non zero positive real numbers, then the division of \( \tilde{A} \) and \( \tilde{B} \) is
\[
\frac{\tilde{A}}{B} = (a_1, a_2, a_3)
\]

(v) For any real number \( K \),
\[
K \tilde{A} = (Ka_1, Ka_2, Ka_3) \text{ if } K > 0
\]
\[
K \tilde{A} = (Ka_3, Ka_2, Ka_1) \text{ if } K < 0
\]

Graded Mean Integration Representation Technique

Defuzzification of \( \tilde{A} \) can be done by Graded Mean Integration Representation Method. If \( \tilde{A} \) is a triangular fuzzy number and is entirely determined by \( (a_1, a_2, a_3) \) then defuzzified value is defined as
\[
G(A) = \frac{1}{2} \int_0^1 h[a_1 + h(a_2 - a_1) + a_3 - h(a_3 - a_2)]dh
\]
\[
= \frac{a_1 + 4a_2 + a_3}{6}
\]

5. MATHEMATICAL MODEL DEVELOPMENT AND ANALYSIS

In this model, we assume that the holding cost increases stepwise as the lot size increases. The following notations are used.

D – Total / Annual Demand
\( \tilde{H} \)- Fuzzy Holding cost per unit
\( \tilde{S}_j \)- Setup cost / Ordering cost per order for the lot size \( \tilde{Q}_j \) if \( q_{j-1} \leq Q_j \leq q_j \)

Where \( j = 1, 2, 3 \ldots m \). \( q_0 = 0 \) and \( q_{\infty} = \infty \)

Also assume \( S_1 < S_2 < S_3 < \ldots \ldots S_m \).

For ordering cost \( \tilde{S}_j \) Harris – Wilson EOQ is given by
\[
\tilde{Q}_j = \sqrt{\frac{2 \times D \times \tilde{S}_j}{\tilde{H}}} \quad \text{------- (3)}
\]

If \( \tilde{Q}_j \) does not lie in the interval \([q_{j-1}, q_j]\) i.e is not order possible, then the optimal lot size will be determined by
\[
q_{j-1} \quad \text{if} \quad Q_j \leq q_{j-1} \quad \text{-------(4)}
\]
\[
q_j \quad \text{if} \quad Q_j \geq q_j \quad \text{-------(5)}
\]
with the well-known value of $\tilde{Q}_j$ thus obtained from the equation (3), (4), and (5), $T\tilde{C}(Q_j)$ can be calculated at the ordering cost by

$$T\tilde{C}(Q_j) = \frac{\tilde{Q}_j \times \tilde{H}}{2} + \frac{D \times \tilde{S}_j}{\tilde{Q}_j}$$

----- (6)

If $\tilde{Q}_j$ is order feasible then $T\tilde{C}(Q_j)$ will be the optimal cost, or else the value of $\tilde{Q}_j$ thus obtained by equations (4) and (5), is calculated by (6). Thus amongst all the calculated values of $T\tilde{C}(Q_j)$, the least rate will be the finest cost. After defuzzification of $\tilde{Q}_j$ and $T\tilde{C}(Q_j)$, we will get optimal TC ($Q_{opt}$) and the corresponding Qj will be the optimal lot size i.e $Q_{opt}$.

6. ALGORITHM

1. Input j, n = number of lot size.
2. Set $j = 1$.
3. Calculate $\tilde{Q}_j = \sqrt{\frac{2 \times D \times \tilde{S}_j}{\tilde{H}}}$
4. If $\tilde{Q}_j$ is order feasible,
5. Calculate $T\tilde{C}(Q_j) = \frac{\tilde{Q}_j \times \tilde{H}}{2} + \frac{D \times \tilde{S}_j}{\tilde{Q}_j}$. Go to Step 8.
6. If $\tilde{Q}_j$ is not order feasible, afterward by equation (4) and (5), $\tilde{Q}_j$ can be calculated and hence find $T\tilde{C}(Q_j)$ by equation (6).
7. Set $j = j + 1$, until $j \leq n$. Go to step 3
8. Defuzzify $\tilde{Q}_j$ as well as $T\tilde{C}(Q_j)$, subsequently we will get $Q_j$ and TC ($Q_j$).
9. Among all the calculated TC ($Q_j$), find the minimum value of TC ($Q_j$), and put TC ($Q_{opt}$) = min [TC ($Q_j$)].
10. Thus TC ($Q_{opt}$) obtained is the best possible cost and consequent $Q_j$ is the $Q_{opt}$ the optimal lot – size.
11. Thus $Q_{opt}$ and TC ($Q_{opt}$) are the necessary results.
**Example 1**

We assume that the hospital need valuable medicines (Demand) from the pharmaceutical company and they spend money (holding cost) for that valuable medicines. 

ie) $D = 10000$ units, $\tilde{H} = (180, 190, 200)$

<table>
<thead>
<tr>
<th>J</th>
<th>Range</th>
<th>$\tilde{S}_j$</th>
<th>$\tilde{Q}_j$</th>
<th>$Q_j$</th>
<th>Feasible $\tilde{Q}_j$</th>
<th>$T\tilde{C}(Q_j)$</th>
<th>$TC(Q_j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>01 – 30</td>
<td>(80, 90, 100)</td>
<td>(89.44, 97.33, 105.41)</td>
<td>97.36</td>
<td>30</td>
<td>(29367, 32850, 36333)</td>
<td>32850</td>
</tr>
<tr>
<td>2</td>
<td>31 – 60</td>
<td>(90, 100, 110)</td>
<td>(94.87, 102.60, 110.55)</td>
<td>102.64</td>
<td>60</td>
<td>(20400, 22367, 24333)</td>
<td>22367</td>
</tr>
<tr>
<td>3</td>
<td>61 – 90</td>
<td>(100, 110, 120)</td>
<td>(100, 107.61, 115.47)</td>
<td>107.65</td>
<td>90</td>
<td>(19211, 20772, 22333)</td>
<td>20772</td>
</tr>
<tr>
<td>4</td>
<td>91 – 120</td>
<td>(110, 120, 130)</td>
<td>(104.88, 112.39, 120.19)</td>
<td>112.44</td>
<td>112.44</td>
<td>(19903, 21354, 22806)</td>
<td>21354</td>
</tr>
<tr>
<td>5</td>
<td>121 – 150</td>
<td>(120, 130, 140)</td>
<td>(109.55, 116.98, 124.72)</td>
<td>117.03</td>
<td>121</td>
<td>(20807, 22239, 23670)</td>
<td>22239</td>
</tr>
</tbody>
</table>

Therefore, Optimal lot size $Q_{opt} = 90$ units and the optimal total cost $TC(Q_{opt}) = Rs. 20772$. 
Example 2

We assume that the hospital need valuable medicines (Demand) from the pharmaceutical company and they spend money (holding cost) for that valuable medicines.

ie) \( D = 1000 \) units, \( \tilde{H} = (190, 200, 210) \)

<table>
<thead>
<tr>
<th>J</th>
<th>Range</th>
<th>( \tilde{S}_j )</th>
<th>( \tilde{Q}_j )</th>
<th>Q</th>
<th>Feasible ( \tilde{Q}_j )</th>
<th>( T\tilde{C}(Q_j) )</th>
<th>TC(Q)</th>
</tr>
</thead>
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<td>(90, 100, 110)</td>
<td>(29.28, 31.62, 34.03)</td>
<td>31.63</td>
<td>20</td>
<td>(6400, 7000, 7600)</td>
<td>7000</td>
</tr>
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<td>2</td>
<td>21 – 30</td>
<td>(100, 110, 120)</td>
<td>(30.86, 33.17, 35.54)</td>
<td>33.18</td>
<td>30</td>
<td>(6183, 6667, 7150)</td>
<td>6667</td>
</tr>
<tr>
<td>3</td>
<td>31 – 40</td>
<td>(110, 120, 130)</td>
<td>(32.37, 34.64, 36.99)</td>
<td>34.65</td>
<td>34.65</td>
<td>(6466, 6928, 7390)</td>
<td>6928</td>
</tr>
<tr>
<td>4</td>
<td>41 – 50</td>
<td>(120, 130, 140)</td>
<td>(33.81, 36.06, 38.39)</td>
<td>36.07</td>
<td>41</td>
<td>(6822, 7271, 7720)</td>
<td>7271</td>
</tr>
<tr>
<td>5</td>
<td>51 - 60</td>
<td>(130, 140, 150)</td>
<td>(35.19, 37.42, 39.74)</td>
<td>37.43</td>
<td>51</td>
<td>(7394, 7845, 8296)</td>
<td>7845</td>
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</table>

Therefore lot size \( Q_{opt} = 30 \) units and the optimal total cost \( TC(Q_{opt}) = Rs. 6667 \).
8. CONCLUSION

In this article, the classical Harris – Wilson model has been extensive with fuzzy ordering cost depending on lot size. It is observed that if order quantity lies within the interval, it will give the best possible costs. Moreover if no such value can be obtained which is order feasible next we can build order feasible by equation (4) and (5). Since all the computed values of the least value will give the best possible ordering costs. We conclude that the best possible cost depends on demand required. The algorithm has been experienced with a numerical example. The results show that the algorithms described in this paper perform well.

We have measured fuzzy nature of ordering/setup cost, holding/carrying cost and order quantity. Even if the total cost in this model appears to be slightly higher than that in the standard model, this model is more suitable to real life situations.

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